Math 474 HW #8

Due 2:00 PM Friday, Nov. 8

1. (Shifrin Problem 2.2.1)

   (a) Suppose Σ is a regular surface with no umbilic points and so that the parameter curves are lines of curvature (i.e., $\vec{x}(t,c)$ and $\vec{x}(c,t)$ are lines of curvature, where t is a variable and c is constant; said otherwise, $\vec{x}_u$ and $\vec{x}_v$ are always principal directions). Show that $F = m = 0$ and that the principal curvatures are $k_1 = \frac{\ell}{E}$ and $k_2 = \frac{n}{G}$.

   (b) On the other hand, suppose Σ is a surface with $F = m = 0$. Prove that the parameter lines are lines of curvature.

2. (Shifrin Problem 2.2.3) Compute the second fundamental form II_p of the following parametrized surfaces. Then calculate the matrix of the shape operator, and determine H and K.

   (a) The cylinder: $\vec{x}(u,v) = (a\cos u, a\sin u, v)$

   (b) The helicoid: $\vec{x}(u,v) = (u\cos v, u\sin v, bv)$

   (c) Enneper's surface: $\vec{x}(u,v) = \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2\right)$