Math 474 HW #7

Due 2:00 PM Friday, Oct. 25

1. Find the first fundamental form (i.e., E, F, and G) of the following surfaces:

   (a) The sphere of radius r: $\vec{x}(u,v) = r(\sin u \cos v, \sin u \sin v, \cos u)$

   (b) The helicoid: $\vec{x}(u,v) = (u \cos v, u \sin v, bv)$.

   (c) The catenoid: $\vec{x}(u,v) = (a \cosh u \cos v, b \cosh u \sin v, u)$.

2. (Shifrin Problem 2.1.6) A parametrization $\vec{x}(u,v)$ of a surface Σ is called *conformal* if angles measured in the uv-plane agree with the corresponding angles in $T_p\Sigma$ for all points $p \in \Sigma$. Prove that the parametrization $\vec{x}(u,v)$ is conformal if and only if $E = G$ and $F = 0$.

3. (Shifrin Problem 2.1.8) Show that the parametrization of the unit sphere by inverse stereographic projection (see HW 6) is conformal.

   Shown below is what you get when you map a square grid in the plane to the sphere by inverse stereographic projection. Notice that all the lines intersect perpendicularly.