Math 474 HW #6

Due 2:00 PM Friday, Oct. 18

1. Consider the sphere $x^2 + y^2 + z^2 = 1$ centered at the origin in $\mathbb{R}^3$. We can construct a very important map called *stereographic projection* from the unit sphere to the xy-plane by defining $\mathrm{st}(p)$ to be the intersection of the line through $p \in S^2$ and $(0,0,1)$ with the xy-plane (of course this map is not well defined at the point $(0,0,1)$).

   (a) Show that the inverse map $\mathrm{st}^{-1} : \mathbb{R}^2 \to S^2 \backslash \{(0,0,1)\}$ is given by

   $$\mathrm{st}^{-1}(u,v) = \left(\frac{2u}{u^2+v^2+1}, \frac{2v}{u^2+v^2+1}, \frac{u^2+v^2-1}{u^2+v^2+1} \right).$$

   (b) Show that st^{-1} is a regular parametrization of $S^2 \backslash \{(0,0,1)\}$.

   (c) Use st^{-1} and the corresponding map $\mathbb{R}^2 \to S^2 \backslash \{(0,0,-1)\}$ to prove (again) that S^2 is a regular surface.