Math 474 HW #4

Due 2:00 PM Friday, Sept. 27

1. Recall that the Bishop frame for a curve $\alpha(s)$ defines associated quantities $k_1(s)$ and $k_2(s)$. The parametrized plane curve $(k_1(s), k_2(s))$ is called the *normal development* of the curve α. Compute the Bishop frame and normal development of a helix.

   (Hint: One strategy [not necessarily the quickest] is to find an arclength parametrization of the helix, use that to compute the Frenet frame of the helix, then write the first Bishop normal vector N_1 as
   $$N_1(s) = \cos(\theta(s))N(s) + \sin(\theta(s))B(s),$$
   where $\theta(s)$ is unknown. Use the fact that $N_1'(s)$ is parallel to $T(s)$ to find an equation for $\theta'(s)$, integrate to find $\theta(s)$, and plug the result into the above equation.)

2. Assume $\alpha(s)$ is an arclength parametrized regular curve with associated Frenet frame (T, N, B). Find a vector $u(s)$ so that
   $$\begin{aligned} T'(s) &= u(s) \times T(s) \\ N'(s) &= u(s) \times N(s) \\ B'(s) &= u(s) \times B(s). \end{aligned}$$
   The vector $u(s)$ is called the *Darboux vector* for α. Find an expression for the length of the Darboux vector in terms of the curvature $\kappa(s)$ and torsion $\tau(s)$ of α.