Math 474 HW #3

Due 2:00 PM Friday, Sept. 20

1. (Shifrin Problem 1.2.8) Let α be a regular curve parametrized by arclength with nonvanishing curvature. The *normal line* to α at $\alpha(s)$ is the line through $\alpha(s)$ with direction vector $N(s)$ (the Frenet normal). Suppose all normal lines to α pass through a given fixed point. What can you say about the curve α? Does your answer change if the curve is not regular?

2. (Shifrin Problem 1.2.11) Suppose $\alpha(t)$ is a regular curve, not necessarily parametrized by arclength. Show that the torsion of α is given by

$$\tau = \frac{\alpha' \cdot (\alpha'' \times \alpha''')}{\|\alpha' \times \alpha''\|^2}.$$

3. (Shifrin Problem 1.2.20) Two distinct parametrized curves α and β are called *Bertrand mates* if, for each t, the normal line to α at $\alpha(t)$ is the same as the normal line to β at $\beta(t)$.

   Suppose α and β are Bertrand mates.

   (a) If α is parametrized by arclength, show that $\beta(s) = \alpha(s) + rN(s)$ for some constant r, meaning that corresponding points on α and β are a constant distance apart.

   (b) Show that, moreover, the angle between the tangent vectors to α and β at corresponding points is constant. (Hint: consider the dot product)

   (c) Suppose α is parametrized by arclength and that both κ and τ are nonvanishing. Show that α has a Bertrand mate β if and only if there are constants r and c so that $r\kappa + c\tau = 1$.

   (d) Show that a curve α that has more than one Bertrand mate must be a helix (and hence have infinitely many Bertrand mates).