# Math 474 HW #2

Due 2:00 PM Friday, Sept. 6

1. (Shifrin Problem 1.1.4) Parametrize the graph $y = f(x)$, $a \leq x \leq b$, and show that its arclength is given by the standard formula

$$\text{length} = \int_a^b \sqrt{1 + (f'(x))^2}\, \mathrm{d}x$$

2. (similar to Shifrin Problem 1.1.8) Let $p, q \in \mathbb{R}^3$ and let $\alpha : [a, b] \to \mathbb{R}^3$ be a parametrized curve such that $\alpha(a) = p$ and $\alpha(b) = q$.

   (a) Show that, for any unit vector $v$,

$$(q - p) \cdot v = \int_a^b \alpha'(t) \cdot v \, \mathrm{d}t \leq \int_a^b \|\alpha'(t)\| \, \mathrm{d}t\,.$$

   (b) Let $v = \frac{q-p}{\|q-p\|}$ and use part (a) to prove that

$$\|q - p\| \leq \text{length}(\alpha).$$

   In other words, the shortest path from $p$ to $q$ is the straight line!

3. (Shifrin Problem 1.2.3(a)–(d), (g)) Compute $(T, \kappa, N, B, \tau)$ for each of the following curves (please use a computer to help you):

   (a) $\alpha(s) = \left(\frac{1}{3}(1+s)^{3/2}, \frac{1}{3}(1-s)^{3/2}, \frac{1}{\sqrt{2}}s\right)$, $s \in (-1, 1)$

   (b) $\alpha(t) = \left(\frac{1}{2}e^t(\sin t + \cos t), \frac{1}{2}e^t(\sin t - \cos t), e^t\right)$

   (c) $\alpha(t) = \left(\sqrt{1+t^2}, t, \ln(t + \sqrt{1+t^2})\right)$

   (d) $\alpha(t) = (e^t \cos t, e^t \sin t, e^t)$

   (e) $\alpha(t) = (t - \sin t \cos t, \sin^2 t, \cos t)$, $t \in (0, \pi)$