

Math 474: Dy 6

Next application:

Prop: A smooth reg curve is spherical $\Leftrightarrow$ its natural development is a line not thru the origin.

Proof ($\Rightarrow$): Suppose α lies on a sphere of center P & radius r . Then

$$(\alpha(s) - P) \cdot (\alpha(s) - P) = r^2$$

Differentiating...

$$\alpha'(s) \cdot (\alpha(s) - P) = 0.$$

so far a Bishop frame $\{N_1, N_2\}$ on $\alpha(s)$,

$$\alpha(s) - P = f(s)N_1(s) + g(s)N_2(s).$$

But now $f'(s) = \frac{d}{ds}(\alpha(s) - P) \cdot N_1(s) = \alpha'(s) \cdot N_1(s) + (\alpha(s) - P) \cdot N_1'(s) = 0$, so f is const. A similar calculation shows g is const as well.

But then $\frac{d}{ds}(\alpha'(s) \cdot (\alpha(s) - P)) = (k_1(s)N_1(s) + k_2(s)N_2(s)) \cdot (\alpha(s) - P) + \alpha'(s) \cdot \alpha'(s) = f k_1(s) + g k_2(s) + 1 = 0$.

But that's the equation of a line!

