

**Math 469 HW #8**  
Due 11:00 PM Sunday, Apr. 12

Assume all vector spaces are finite-dimensional.

1. (Axler Problem 7.D.14) Suppose  $T \in \mathcal{L}(V)$ . Prove that  $\dim \operatorname{range} T$  equals the number of nonzero singular values of  $T$ .
2. (Axler Problem 7.D.16) Suppose  $T_1, T_2 \in \mathcal{L}(V)$ . Prove that  $T_1$  and  $T_2$  have the same singular values if and only if there exist isometries  $S_1, S_2 \in \mathcal{L}(V)$  so that  $T_1 = S_1 T_2 S_2$ .
3. (Axler Problem 8.A.9) Suppose  $S, T \in \mathcal{L}(V)$  and  $ST$  is nilpotent. Prove that  $TS$  is also nilpotent.