Math 469 HW #7

Due 11:00 PM Sunday, Apr. 5

1. (Axler Problem 7.A.21) Fix a positive integer n. In the inner product space of continuous real-valued functions on $[-\pi, \pi]$ with inner product

$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)\,\mathrm{d}x,$$

let

$$V = \operatorname{span}(1, \cos(x), \cos(2x), \ldots, \cos(nx), \sin(x), \sin(2x), \ldots, \sin(nx)).$$

   (a) Define $D \in \mathcal{L}(V)$ by $Df = f'$. Show that $D^* = -D$. Conclude that D is normal but not self-adjoint.

   (b) Define $T \in \mathcal{L}(V)$ by $Tf = f''$. Show that T is self-adjoint.

2. (Axler Problem 7.B.2) Suppose that T is a self-adjoint operator on a finite-dimensional inner product space and that 2 and 3 are the only eigenvalues of T. Prove that $T^2 - 5T + 6I = 0$.

3. (Axler Problem 7.C.14) Let T be the second derivative operator from Problem 1(b) above. Show that $-T$ is a positive operator.