Math 469 HW #6

Due 1:00 PM Friday, Mar. 13

1. (Axler Problem 5.A.22) Suppose $T \in \mathcal{L}(V)$ and there exist nonzero vectors $v, w \in V$ so that
$$Tv = 3w \quad \text{and} \quad Tw = 3v.$$
Prove that 3 or -3 is an eigenvalue of T.

2. (Axler Problem 6.A.12) Prove that
$$(x_1 + \cdots + x_n)^2 \leq n(x_1^2 + \cdots + x_n^2)$$
for all positive integers n and all real numbers $x_1, \ldots, x_n$.

3. (Axler Problem 6.A.24) Suppose $S \in \mathcal{L}(V)$ is injective and that V comes equipped with some inner product $\langle \cdot, \cdot \rangle$. Define a new inner product $\langle \cdot, \cdot \rangle_S$ by
$$\langle u, v \rangle_S = \langle Su, Sv \rangle$$
for all $u, v \in V$. Show that $\langle \cdot, \cdot \rangle_S$ is an inner product on V.

4. (Axler Problem 6.b.2) Suppose $e_1, \ldots, e_m$ is an orthonormal list of vectors in V. Let $v \in V$. Prove that
$$\|v\|^2 = |\langle v, e_1 \rangle|^2 + \cdots + |\langle v, e_m \rangle|^2$$
if and only if $v \in \operatorname{span}(e_1, \ldots, e_m)$