Math 469 HW #4

Due 1:00 PM Friday, Feb. 21

1. (Axler Problem 3.C.3) Suppose V and W are finite-dimensional and $T \in \mathcal{L}(V, W)$. Prove that there exist a basis of V and a basis of W so that, with respect to these bases, all entries of $\mathcal{M}(T)$ are 0 except that the entries in row j, column j equal 1 for $1 \leq j \leq \dim \operatorname{range} T$.

2. (Axler Problem 3.D.2) Suppose v is finite-dimensional and $\dim V > 1$. Prove that the set of noninvertible operators on V is not a subspace of $\mathcal{L}(V)$.

3. (Axler Problem 3.D.4) Suppose W is finite-dimensional and $T_1, T_2 \in \mathcal{L}(V, W)$. Prove that $\operatorname{null} T_1 = \operatorname{null} T_2$ if and only if there exists an invertible operator $S \in \mathcal{L}(W)$ so that $T_1 = ST_2$.

4. (Axler Problem 3.D.5) Suppose V is finite-dimensional and $T_1, T_2 \in \mathcal{L}(V, W)$. Prove that $\operatorname{range} T_1 = \operatorname{range} T_2$ if and only if there exists an invertible operator $S \in \mathcal{L}(V)$ so that $T_1 = T_2 S$.