Math 469 HW #3

Due 1:00 PM Friday, Feb. 14

1. (Axler Problem 2.C.15) Suppose V is finite-dimensional, with $\dim V = n \geq 1$. Prove that there exist 1-dimensional subspaces $U_1, \dots, U_n$ of V so that

$$V = U_1 \oplus \cdots \oplus U_n.$$

2. (Axler Problem 3.A.7) Show that every linear map from a 1-dimensional vector space to itself is multiplication by some scaler. More precisely, prove that if $\dim V = 1$ and $T \in \mathcal{L}(V, V)$, then there exists some $\lambda \in \mathbb{F}$ so that $Tv = \lambda v$ for all $v \in V$.

3. (Axler Problem 3.A.11) Suppose V is finite-dimensional. Prove that every linear map on a subspace of V can be extended to a linear map on V. In other words, show that if U is a subspace of V, W is some vector space, and $S \in \mathcal{L}(U, W)$, then there exists $T \in \mathcal{L}(V, W)$ so that $T(u) = S(u)$ for all $u \in U$.

4. (Axler Problem 3.B.5) Give an example of a linear map $T : \mathbb{R}^4 \to \mathbb{R}^4$ so that $\operatorname{range} T = \operatorname{null} T$.