Math 469 HW #2

Due 1:00 PM Friday, Feb. 7

1. (Axler Problem 2.A.9) Prove or give a counterexample: If $v_1, \ldots, v_m$ and $w_1, \ldots, w_m$ are linearly independent lists of vectors in V, then $v_1 + w_1, \ldots, v_m + w_m$ is linearly independent.

2. (Axler Problem 2.A.14) Prove that V is infinite-dimensional if and only if there is a sequence $v_1, v_2, \ldots$ of vectors in V so that $v_1, \ldots, v_m$ is linearly independent for every positive integer m.

3. (Axler Problem 2.B.5) Prove or disprove: There exists a basis p_0, p_1, p_2, p_3 of $\mathcal{P}_3(\mathbb{F})$ so that none of the polynomials p_0, p_1, p_2, p_3 has degree 2.

4. (Axler Problem 2.B.8) Suppose U and W are subspaces of V so that $V = U \oplus W$. Suppose also that $u_1, \ldots, u_m$ is a basis of U and $w_1, \ldots, w_n$ is a basis of W. Prove that

$$u_1, \ldots, u_m, w_1, \ldots, w_n$$

   is a basis of V.