Math 419 HW #9

Due 3:00 PM Friday, Nov. 15

1. (Jones Problem 5–4 (1.))

   (a) Let
   $$f(z) = 1 - e^{-z}.$$
   Notice that $f(0) = 0$ and $f'(0) = 1 \neq 0$, so f has a holomorphic inverse near 0. Use the Lagrange–Bürmann expansion with $g(z) = z$ to find a power series centered at 0 for $f^{-1}(w)$; the coefficients should be expressed in terms of residues. (You don't have to compute the residues.)

   (b) Use high school algebra to explicitly write down an equation for $f^{-1}(w)$.

   (c) Use parts (a) and (b) to compute
   $$\operatorname{Res}\left(\frac{1}{(1-e^{-z})^n}, 0\right).$$

2. Let
   $$f(z) = \frac{z}{\sqrt{1+z}}.$$
   Prove that we have the following Taylor series centered at 0 for $f^{-1}(w)$:
   $$f^{-1}(w) = \sum_{n=1}^{\infty} c_n w^n = w + \frac{w^2}{2} + \frac{w^3}{3} - \frac{w^5}{128} + \frac{w^7}{1024} - \frac{5w^9}{32{,}768} + \dots,$$
   where
   $$c_n = \begin{cases} 1 & \text{if } n = 1 \\ \frac{1}{2} & \text{if } n = 2 \\ (-1)^{\frac{n+1}{2}} \frac{(n-2)(n-4)^2 \cdots 3^2 \cdot 1^2}{(n-1)! 2^{n-1}} & \text{if } n > 1 \text{ is odd} \\ 0 & \text{if } n > 1 \text{ is even.} \end{cases}$$