Math 419 HW #6

Due 3:00 PM Friday, Oct. 18

1. Let $D \subset \mathbb{C}$ be an open set and let γ be a circle contained in D. Suppose f is holomorphic on D except possibly at a point z_0 inside γ. Prove that if f is bounded near z_0, then

$$\int_\gamma f(z)\,\mathrm{d}z = 0.$$

2. The function $f(z) = e^{1/z}$ has an essential singularity at $z = 0$. Verify the truth of Picard's great theorem for f. In other words, show that for any $w \in \mathbb{C}$ (with possibly one exception) there is a sequence $z_1, z_2, \ldots$ with $z_k \to 0$ and $f(z_k) = w$ for all k.