Math 419 HW #5

Due 3:00 PM Friday, Oct. 4

1. Let γ be the unit circle and evaluate the integrals

   (a) $\displaystyle\int_\gamma \frac{e^z}{z^2}\,\mathrm{d}z$

   (b) $\displaystyle\int_\gamma \frac{\sin z}{z^4}\,\mathrm{d}z$

2. (Jones Problem 3–1) There is a unique Möbius transformation f of $\widehat{\mathbb{C}}$ which satisfies

$$f(0) = -1$$
$$f(\infty) = 1$$
$$f(i) = 0.$$

   This Möbius transformation is sometimes called the *Caley transformation.*

   (a) Find a, b, c, d so that $f(z) = \frac{az+b}{cz+d}$.

   (b) Show that $f(\mathbb{R} \cup \{\infty\}) =$ the unit circle.

   (c) Show that f(open upper half plane) = the open unit disk (i.e., the region inside the unit circle).

   (d) For several $y > 0$, sketch the image of the horizontal line $\{x+iy | x \in \mathbb{R}\}$ under the action of f.