Math 419 HW #3

Due 3:00 PM Friday, Sept. 20

1. With $z = re^{i\theta}$ with $r \geq 0$ and n a positive integer, solve the equation $w^n = z$. How many solutions are there?

2. The Cauchy–Riemann equation
$$\frac{\partial f}{\partial x} = \frac{1}{i}\frac{\partial f}{\partial y},$$
which is satisfied by any complex-differentiable function, motivates us to define two differential operators
$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) \qquad \text{and} \qquad \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right).$$
Notice that f satisfies the Cauchy–Riemann equation if and only if $\frac{\partial f}{\partial \bar{z}} = 0$, and that if f is complex-differentiable then $\frac{df}{dz} = \frac{\partial f}{\partial z}$.
Show that
$$4\frac{\partial}{\partial z}\frac{\partial}{\partial \bar{z}} = 4\frac{\partial}{\partial \bar{z}}\frac{\partial}{\partial z} = \Delta,$$
where Δ is the *Laplacian*
$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

3. (Jones Exercise on p. 26) Prove that the equation
$$\frac{\partial f}{\partial r} = \frac{1}{ir}\frac{\partial f}{\partial \theta} \tag{1}$$
implies the original Cauchy–Riemann equation. In other words, Jones shows that if f satisfies the Cauchy–Riemann equation, then it satisfies (1); your goal in this problem is to show that if f satisfies (1), then it satisfies the Cauchy–Riemann equation. These two results together imply that the Cauchy–Riemann equation and (1) are equivalent, so it is sensible to call (1) the polar Cauchy–Riemann equation.