Math 369: Day 4

Last time, we started talking about vectors, which we saw we can add & scale. But then...

Ex: The system of linear equations

$$\begin{aligned} 2x_1 + 3x_2 - 5x_3 &= 17 \\ x_1 - x_2 + x_3 &= 1 \end{aligned}$$

Has exactly the same solutions x_1, x_2, x_3 as the single vector equation

$$x_1\begin{pmatrix}2\\1\end{pmatrix} + x_2\begin{pmatrix}3\\-1\end{pmatrix} + x_3\begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}17\\1\end{pmatrix}.$$

In other words, this equation says that the vector $\begin{pmatrix}17\\1\end{pmatrix}$ can be written as a linear combination of the vectors $\begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}3\\-1\end{pmatrix}, \begin{pmatrix}-5\\1\end{pmatrix}$.

Matrices:

Def: If m, n are natural numbers, then $Mat_{m,n}(\mathbb{R})$ is the set of matrices w/ real entries w/ m rows & n columns.

Ex: $Mat_{2,3}(\mathbb{R}) = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} : a_{ij} \in \mathbb{R} \right\}$

As w/ vectors, we can add 2 matrices of the same shape and can scale matrices.

Moreover, we can multiply a matrix times a vector:

Ex: $\begin{pmatrix} 2 & 3 & -5 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix} 2x_1 + 3x_2 - 5x_3 \\ x_1 - x_2 + x_3 \end{pmatrix}$

In general, if $A = (a_{ij}) \in Mat_{m,n}(\mathbb{R})$ & $\vec{v} = \begin{pmatrix}v_1\\ \vdots \\ v_n\end{pmatrix} \in \mathbb{R}^n$, then

$$A\vec{v} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}\begin{pmatrix}v_1\\ \vdots \\ v_n\end{pmatrix} = \begin{pmatrix} a_{11}v_1 + \cdots + a_{1n}v_n \\ \vdots \\ a_{m1}v_1 + \cdots + a_{mn}v_n \end{pmatrix}$$

Notice that if $A = \begin{pmatrix} 2 & 3 & -5 \\ 1 & -1 & 1 \end{pmatrix}$, $\vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ & $\vec{b} = \begin{pmatrix}17\\1\end{pmatrix}$, then the matrix equation

$A\vec{x} = \vec{b}$ is just $\begin{pmatrix} 2 & 3 & -5 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}17\\1\end{pmatrix}$

$\begin{pmatrix} 2x_1 + 3x_2 - 5x_3 \\ x_1 - x_2 + x_3 \end{pmatrix} = \begin{pmatrix}17\\1\end{pmatrix}$ or $x_1\begin{pmatrix}2\\1\end{pmatrix} + x_2\begin{pmatrix}3\\-1\end{pmatrix} + x_3\begin{pmatrix}-5\\1\end{pmatrix} = \begin{pmatrix}17\\1\end{pmatrix}$,

which we already said is the same as the system of linear eqns $\begin{aligned} 2x_1 + 3x_2 - 5x_3 &= 17 \\ x_1 - x_2 + x_3 &= 1 \end{aligned}$

So the point is that we can write any system of linear eqns as a single matrix eqn of the form

$$A\vec{x} = \vec{b}$$

If A, $\vec{x}$ & $\vec{b}$ were numbers, how would you solve for $\vec{x}$?

Divide by A... or, more precisely, multiply by A^{-1}.

Well, that's a strategy for solving matrix eqns as well once we figure out what "multiply by A^{-1}" means.