Math 369: Day 15

Return Exam 1

Score breakdown:

Score Range	Grade
21–23	A
18–20	B
15–17	C
≤ 14	D/F

Comments:

#1: Be careful: just b/c there are free variables doesn't mean there are infinitely many solutions, & just b/c there are the same # of variables & equations doesn't mean there is a unique solution.

#2: If you don't know what a linear combination is, learn fast.

#4: Need to show additivity & homogeneity in general, not just for specific examples.

#5: Interesting that nobody suggested the answer might be -4.

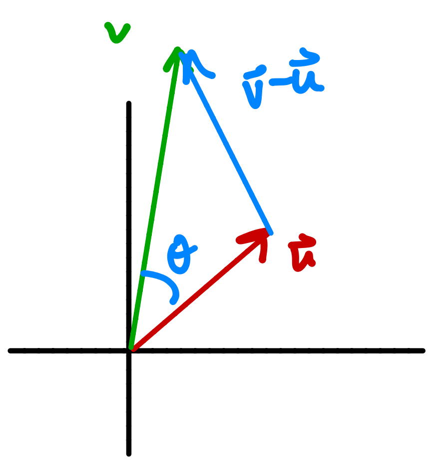
Recall:

if $\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} \in \mathbb{R}^n$, then $\|\vec{v}\| = \sqrt{v_1^2 + \dots + v_n^2}$.

the dot product of 2 vectors to be $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + \dots + u_nv_n$ if $\vec{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ & $\vec{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$

if $\vec{u}, \vec{v} \in \mathbb{R}^n$ s.t. $\vec{u} \cdot \vec{v} = 0$, then $\vec{u}$ & $\vec{v}$ are perpendicular.

More generally,



By the law of cosines, $\|\vec{v}-\vec{u}\|^2 = \|\vec{u}\|^2 + \|\vec{v}\|^2 - 2\|\vec{u}\|\|\vec{v}\| \cos\theta$

$\updownarrow$ equal

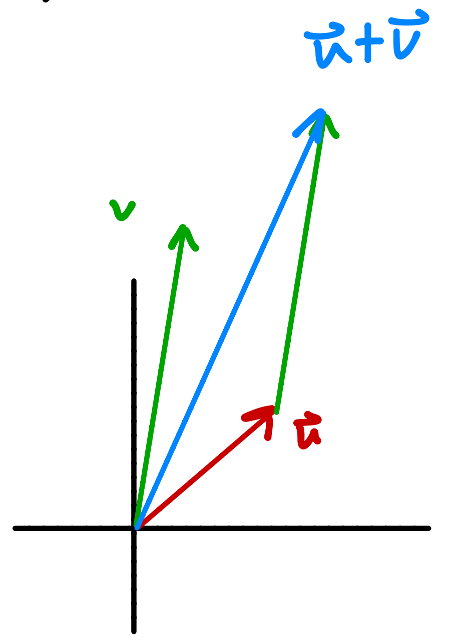
But then $\|\vec{v}-\vec{u}\|^2 = (\vec{v}-\vec{u}) \cdot (\vec{v}-\vec{u}) = \vec{v} \cdot \vec{v} - 2\vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{u} = \|\vec{v}\|^2 - 2\vec{u} \cdot \vec{v} + \|\vec{u}\|^2$

So we see that $\cancel{\|\vec{u}\|^2} + \cancel{\|\vec{v}\|^2} - 2\|\vec{u}\|\|\vec{v}\| \cos\theta = \cancel{\|\vec{v}\|^2} - 2\vec{u} \cdot \vec{v} + \cancel{\|\vec{u}\|^2}$, meaning $2\|\vec{u}\|\|\vec{v}\| \cos\theta = 2\vec{u} \cdot \vec{v}$,

So $\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos\theta$.

Thm (Cauchy-Schwarz): If $\vec{u}, \vec{v} \in \mathbb{R}^n$, then $|\vec{u} \cdot \vec{v}| \le \|\vec{u}\| \|\vec{v}\|$.

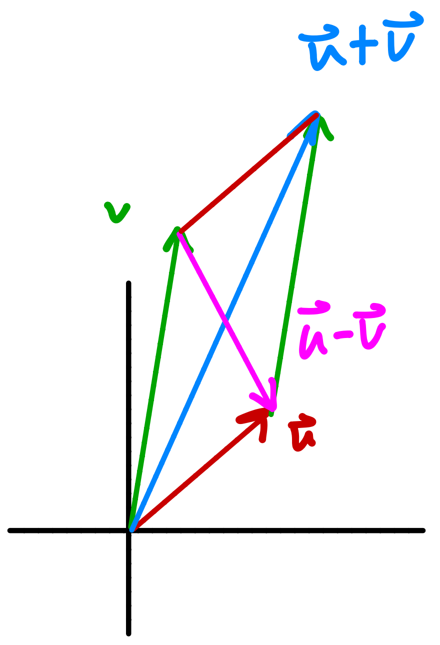
Thm (Triangle Inequality) For vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$, $\|\vec{u}+\vec{v}\| \leq \|\vec{u}\|+\|\vec{v}\|$.



Clearly, the length of any one side of a triangle can be no more than the sum of the lengths of the other two sides.

In fact, it's clear from the picture that we only get $\|\vec{u}+\vec{v}\| = \|\vec{u}\|+\|\vec{v}\|$ when $\vec{u}$ & $\vec{v}$ are parallel.

Thm (Parallelogram Law): For vectors $\vec{u}, \vec{v} \in \mathbb{R}^n$, $\|\vec{u}+\vec{v}\|^2 + \|\vec{u}-\vec{v}\|^2 = 2(\|\vec{u}\|^2 + \|\vec{v}\|^2)$



Proof: $\|\vec{u}+\vec{v}\|^2+\|\vec{u}-\vec{v}\|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) + (\vec{u}-\vec{v})\cdot(\vec{u}-\vec{v}) = \vec{u}\cdot\vec{u} + \vec{v}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v} + \vec{u}\cdot\vec{u} - \vec{v}\cdot\vec{u} - \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}$

$$= 2(\vec{u}\cdot\vec{u} + \vec{v}\cdot\vec{v})$$

$$= 2(\|\vec{u}\|^2 + \|\vec{v}\|^2)$$

Q.E.D.