Math 369 HW #9
Due 8:00 AM Friday, Apr. 14

1. Suppose a, b, c, d are integers so that $a + b = c + d$. Show that the matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

 has integer eigenvalues.

2. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

 Find a diagonal matrix D and an invertible matrix P so that $A = PDP^{-1}$.

3. Let

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

 (a) Find the eigenvalues of B.

 (b) For each eigenvalue λ, find the rank of the matrix $B - \lambda I$.

 (c) Is B diagonalizable? Explain.

4. Let

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

 (a) Diagonalize A. In other words, find P and D so that $A = PDP^{-1}$.

 (b) Use your answer to part (a) to compute A^{10}.

5. Find A^n if n is a positive integer and

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$