Math 369 HW #7

Due 8:00 AM Friday, Mar. 24

1. (a) Show that the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 3 \end{bmatrix}$ form a linearly dependent set in $\mathbb{R}^4$.

   (b) Express each vector in (a) as a linear combination of the other two.

2. Determine which of the following sets of elements of $F(-\infty, \infty)$ are linearly dependent. The following trig identities may come in handy:

$$\sin^2 x + \cos^2 x = 1, \quad \sin(2x) = 2 \sin x \cos x, \quad \cos(2x) = \cos^2 x - \sin^2 x.$$

   (a) $\{6, 3\sin^2 x, 2\cos^2 x\}$

   (b) $\{x, \cos x\}$

   (c) $\{1, \sin x, \sin(2x)\}$

   (d) $\{\cos(2x), \sin^2 x, \cos^2 x\}$

   (e) $\{(3 - x)^2, x^2 - 6x, 5\}$

   (f) $\{0, \cos^3(\pi x), \sin^5(3\pi x)\}$

3. Let $\vec{u}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. In each of the following, find the coordinates of $\vec{w}$ with respect to the basis $\{\vec{u}_1, \vec{u}_2\}$ of $\mathbb{R}^2$.

   (a) $\vec{w} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

   (b) $\vec{w} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

4. Recall that P_2 is the vector space of polynomials of degree ≤ 2; i.e., all polynomials of the form $ax^2 + bx + c$ for any numbers a, b, c. Let $p_1 = x^2 + 2x + 1$, $p_2 = 9x + 2$, $p_3 = 4x^2 + 3x + 3$.

   (a) Show that $\{p_1, p_2, p_3\}$ is a basis for P_2.

   (b) Find the coordinates for $p = -3x^2 + 17x + 2$ with respect to the basis $\{p_1, p_2, p_3\}$.

5. In each part, find a basis for the given subspace of $\mathbb{R}^4$, and state its dimension.

   (a) All vectors of the form $\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix}$

   (b) All vectors of the form $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, where $a = b = c = d$.

(c) All solutions of the equation

$$\begin{bmatrix} 5 & 7 & 1 & -6 \\ 1 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$