Math 369 HW #6

Due 8:00 AM Friday, Mar. 10

1. For each of the following, say whether the given collection is a vector space or not. If the collection is not a vector space show that it fails one of the axioms.

   (a) The set of all 2-dimensional vectors of the form $\begin{bmatrix} x \\ 0 \end{bmatrix}$ with the usual operations in $\mathbb{R}^2$.

   (b) The set of all 2×2 invertible matrices with the standard matrix addition and scalar multiplication.

   (c) The set of all real-valued functions f defined on the whole real line with the property that $f(1) = 0$. The operations are the usual function addition and scalar multiplication.

2. Suppose ℓ is a line in the plane $\mathbb{R}^2$ and consider the collection of all vectors lying in ℓ.

   (a) Show that if ℓ passes through the origin then this collection is a vector space.

   (b) Show that if ℓ does *not* pass through the origin then this collection fails at least one of the vector space axioms.

3. Which of the following are subspaces of $F(-\infty, \infty)$?

   (a) All functions f in $F(-\infty, \infty)$ with $f(0) = 0$.

   (b) All functions f in $F(-\infty, \infty)$ with $f(0) = 1$.

   (c) All functions f in $F(-\infty, \infty)$ with $f(-x) = f(x)$ for all x.

   (d) All polynomials of degree ≤ 2.

4. For each of the following, write the given vector as a linear combination of $\vec{u} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$.

   (a) $\begin{bmatrix} -9 \\ -7 \\ -15 \end{bmatrix}$

   (b) $\begin{bmatrix} 6 \\ 11 \\ 6 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

5. In each part, let $T_A : \mathbb{R}^3 \to \mathbb{R}^2$ be multiplication by the matrix A, and let $\vec{u}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, and $\vec{u}_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$. Determine whether the set $\{T_A(\vec{u}_1), T_A(\vec{u}_2), T_A(\vec{u}_3)\}$ spans $\mathbb{R}^2$:

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & -3 \end{bmatrix}$