Math 369 HW #5

Due 8:00 AM Friday, Mar. 3

1. For each of the following pairs of vectors, determine the cosine of the angle between the two vectors. Is the angle acute, obtuse, or right?

   (a) $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ -3 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 5 \\ 1 \\ 2 \\ -2 \end{bmatrix}$ (b) $\vec{u} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \\ 3 \end{bmatrix}$

2. Let $\vec{p} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$ be a fixed vector in $\mathbb{R}^3$. For each part, describe in words the set of all vectors $\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ satisfying the stated equation.

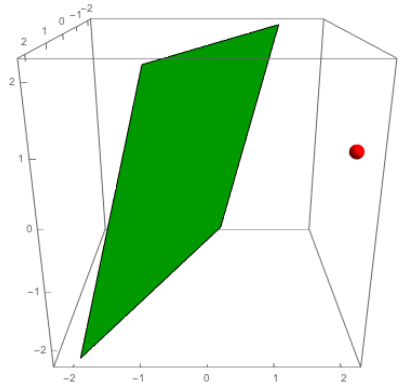
   (a) $\|\vec{v} - \vec{p}\| = 1$

   (b) $\|\vec{v} - \vec{p}\| \leq 1$

   (c) $\|\vec{v} - \vec{p}\| > 1$

3. Let $\vec{u} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$. Write $\vec{u}$ as the sum of two vectors: one parallel to $\vec{v}$ and one perpendicular to $\vec{v}$.

4. Let P be the plane $2x + 4y - z + 1 = 0$ and let Q be the point $(2, 2, 1)$ shown in red in the figure. What is the distance from Q to P?



5. Suppose the vector $\vec{v} \in \mathbb{R}^n$ is perpendicular to both $\vec{w}_1$ and $\vec{w}_2$. Show that $\vec{v}$ is also perpendicular to any vector of the form $a\vec{w}_1 + b\vec{w}_2$ for any numbers a and b.