Math 369 HW #3

Due 8:00 AM Friday, Feb. 10

1. Let A be the 3×3 matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}.$$

   Then A induces a linear transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$. What is the range of T_A?

2. For each of the following, determine whether the given function is a linear transformation. If it *is* a linear transformation, prove it (for example, by showing that it satisfies the conditions in the definition of a linear transformation); if it *isn't* a linear transformation, give a specific example that shows that some part of the definition fails.

   (a) $f : \mathbb{R}^3 \to \mathbb{R}^2$ given by

   $$f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} \right) = \begin{bmatrix} 4x - y \\ 3y \end{bmatrix}$$

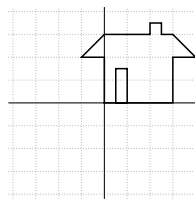   (b) $g : \mathbb{R}^2 \to \mathbb{R}^2$ given by

   $$g\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} 2x + 3y \\ -3xy \end{bmatrix}$$

3. Does there exist a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that

   $$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \qquad \text{and} \qquad T\left(\begin{bmatrix} 5 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 12 \\ 2 \end{bmatrix}?$$

   Explain your answer.
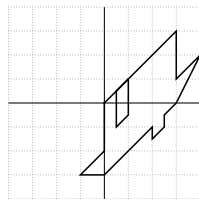
4. Consider the following figure:

   (a) Sketch the figure which results after applying the following linear transformation to the above figure:

   $$R\left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} \cos \pi/3 & \sin \pi/3 \\ -\sin \pi/3 & \cos \pi/3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

   (b) Describe the linear transformation S which, when applied to the above figure, produces:

(For example, finding the matrix representation of S would be a good description.)

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which rotates objects in the plane around the origin by 30 degrees counterclockwise.

   (a) Give the standard matrix A for the transformation T.

   (b) Use your answer to part (a) to compute $T\left(\begin{bmatrix} 5\sqrt{3}/2 \\ 7/2 \end{bmatrix}\right)$.