Math 369 HW #2

Due 8:00 AM Friday, Feb. 3

1. Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 & 1 \\ 3 & -4 \\ -4 & -2 \end{pmatrix}$$

   (a) Compute AB.
   (b) Compute BA.
   (c) Write each column of AB as a linear combination of the columns of A.
   (d) Write each column of BA as a linear combination of the columns of B.

2. For any $n \times n$ matrix A and any positive integer m, define the matrix power $A^m = \underbrace{A \cdot A \cdots A}_{m \text{ times}}$.

   Let $D = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$ and let $U = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$.

   (a) Compute D^2 and D^3.
   (b) What is D^m for general m?
   (c) Compute U^2 and U^3.
   (d) What is U^m for general m?

3. (1.3.30) Let $\mathbf{0}$ be the 2×2 matrix with all zero entries.

   (a) Does there exist a 2×2 matrix A so that $A \neq \mathbf{0}$, but $AA = \mathbf{0}$? Justify your answer.
   (b) Does there exist a 2×2 matrix A so that $A \neq \mathbf{0}$ and $AA = A$? Justify your answer.

4. (1.4.6 and 1.4.28) For each of the following matrices, determine whether it is invertible and, if it is, compute its inverse.

   (a) $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

5. Suppose that $\vec{u}$ and $\vec{v}$ are solutions to the homogeneous equation

$$A\vec{x} = \vec{0}.$$

   Show that $\vec{u} + \vec{v}$ is also a solution to this equation.