Math 369 HW #11

Due 8:00 AM Friday, May 5

1. Which of the following functions define legitimate inner products on $\mathbb{R}^3$? In each of the following, $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ are arbitrary elements of $\mathbb{R}^3$. If the function *is* an inner product just say so, if it is *not* an inner product, give an explicit example where one of the axioms fails.

   (a) $\langle \vec{u}, \vec{v} \rangle = u_1 v_1 + u_3 v_3$.

   (b) $\langle \vec{u}, \vec{v} \rangle = u_1 v_1 - u_2 v_2 + u_3 v_3$.

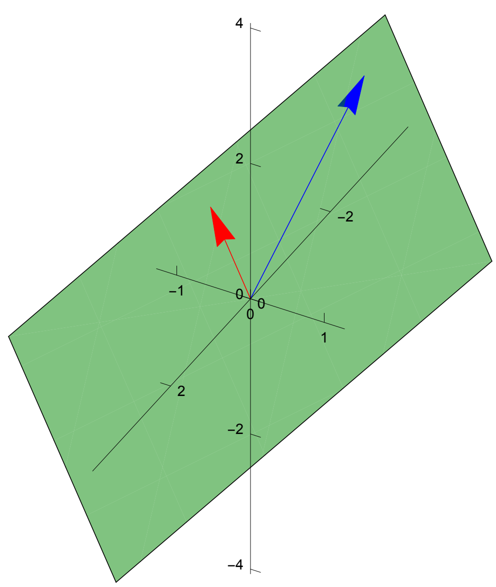
   (c) $\langle \vec{u}, \vec{v} \rangle = 2u_1 v_1 + u_2 v_2 + 4u_3 v_3$.

   (d) $\langle \vec{u}, \vec{v} \rangle = u_1 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$.

2. (a) Show that $\mathcal{B} = \left\{ \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{bmatrix}, \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \right\}$ is an orthonormal basis for $\mathbb{R}^3$ with respect to the standard dot product (orthonormal sets are automatically linearly independent and $\mathbb{R}^3$ is 3-dimensional, so it is enough just to show that $\mathcal{B}$ is an orthonormal set).

   (b) Write the vector $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ as a linear combination of the basis vectors. Equivalently, determine $[\vec{u}]_{\mathcal{B}}$.

Shown below are the vectors $\vec{u} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ (red) and $\vec{v} \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ (blue). Also shown is the (green) plane P that they span. $\vec{u}, \vec{v}$, and P will be used in problems 3 and 4.



3. Apply the Gram-Schmidt procedure to $\{\vec{u}, \vec{v}\}$ to find an orthonormal basis for P.

4. Use your answer to Problem #3 to determine the projection of the vector $\vec{w} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ onto P.

5. After the zombie apocalypse, a group of CSU students bands together to install remote zombie sensors around Fort Collins. After 11 days, the sensors say that there are 1317 zombies in Fort Collins. After 18 days there are 8991, after 33 days there are 10,553, and after 44 days there are 13,873 zombies. Use a linear model to predict the zombie population after 75 days.

   In other words, find the line $z(t) = mt + b$ which best fits the data; then the $z(t)$ function is the linear approximation to the number of zombies in Fort Collins after t days, so you can just evaluate $z(75)$.

   (Obviously, the calculation is going to be very painful if you don't use a calculator/computer.)