Math 369 HW #10

Due 8:00 AM Friday, Apr. 21

1. Consider the stochastic matrix

$$P = \begin{bmatrix} 1/3 & 1/4 & 2/5 \\ 0 & 3/4 & 2/5 \\ 2/3 & 0 & 1/5 \end{bmatrix}.$$

   What is the steady-state vector for the associated Markov chain?

2. Consider a Markov process with transition matrix

$$\begin{bmatrix} 0 & 1/7 \\ 1 & 6/7 \end{bmatrix}$$

   (a) What does the entry 6/7 represent?

   (b) What does the entry 0 represent?

   (c) If the system is in state 1 initially, what is the probability that it will be in state 1 at the next observation?

   (d) If the system has a 50% chance of being in state 1 initially, what is the probability that it will be in state 2 at the next observation?

3. (a) Solve the system

$$\begin{aligned} y_1' &= y_1 + 3y_2 \\ y_2' &= 4y_1 + 5y_2. \end{aligned}$$

   (b) Find the solution to the system that satisfies the initial conditions $y_1(0) = 2$ and $y_2(0) = 1$.

4. Sometimes it is possible to solve a single higher-order differential equation by transforming it into a system of first-order equations. Consider the differential equation

$$y'' + y' - 12y = 0.$$

   Let $y_1 = y$ and let $y_2 = y'$.

   (a) Re-write the above second-order equation as a system of two first-order equations of the form

$$\begin{aligned} y_1' &= ay_1 + by_2 \\ y_2' &= cy_1 + dy_2 \end{aligned}$$

   for some appropriate choice of a, b, c, d.

   (b) Solve the system you derived in part (a).

   (c) Use your solution from part (b) to solve the original equation $y'' + y' - 12y = 0$.

5. Let $C^\infty(-\infty, \infty)$ be the vector space of infinitely-differentiable functions defined on all of $\mathbb{R}$. In other words, these are the functions that we can differentiate as many times as we like. This is clearly a vector space, since the sum of two differentiable functions is differentiable and a constant times a differentiable functions is differentiable.

   Define the mapping $L : C^\infty(-\infty, \infty) \to C^\infty(-\infty, \infty)$ by

   $$L(f) = f'' + 2f' - 3f.$$

   (a) Show that L is a linear transformation.

   (b) Determine the nullspace of L. In other words, determine all f so that $L(f) = 0$.