

### Math 369 Exam #2 Practice Problems

1. Consider the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$  given by

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2x - 3y \\ x + y \\ 2x \end{pmatrix}.$$

- (a) Show that  $f$  is a linear transformation.  
 (b) Is  $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$  in the image of  $f$ ?
2. Consider the matrix

$$A = \begin{pmatrix} -3 & -4 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

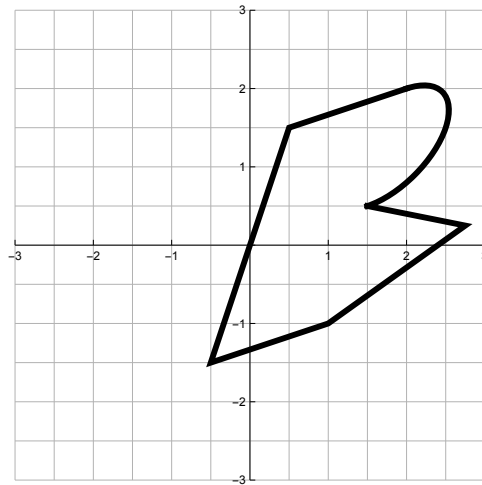
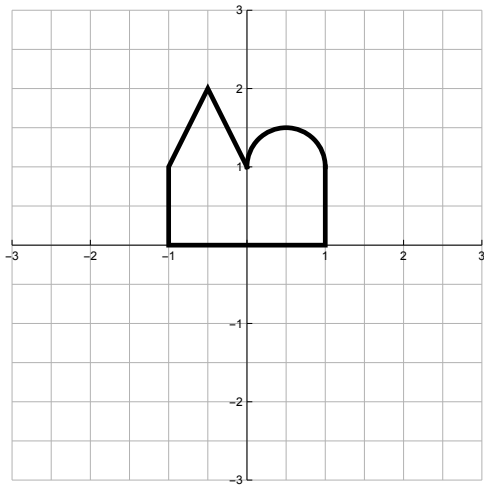
Find the characteristic polynomial of  $A$ .

3. Logan and Terry are both computing with the same  $5 \times 3$  matrix. Logan determines that the nullspace of the matrix is 2-dimensional, while Terry computes that the column space is 2-dimensional. Can they both be right? Justify your answer.
4. Suppose  $V$  is a vector space and that  $f : V \rightarrow V$  is a linear transformation. Let

$$V_\lambda = \{v \in V : f(v) = \lambda v\}.$$

Show that  $V_\lambda$  is a subspace of  $V$ .

5. Consider the following images:



The figure on the right shows the image of the figure on the left under the action of the linear transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ .

- (a) What is the matrix for  $T$  with respect to the standard basis on  $\mathbb{R}^2$ ? In other words, what is  $[T]_{\text{std}_2 \rightarrow \text{std}_2}$ ?
- (b) What are  $T\left(\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right)$  and  $T\left(\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right)$ ?
- (c) Let  $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$ . What is  $[T]_{\mathcal{B} \rightarrow \mathcal{B}}$ ? (*Hint: Use (b)...*)