Math 336 HW #8

Due 5:00 PM Tuesday, April 13

Presentations:

1. Show, starting from the theorem that $g(K\#K') = g(K) + g(K')$, that any knot can be expressed as a finite connect sum of prime knots.

Problems:

1. Prove that the Figure 8 knot is equivalent to its mirror.
2. Define an n-coloring of a knot diagram D combinatorially ($n \geq 3$). That is, derive conditions at a crossing that will make the number of nontrivial n-colorings of a knot an invariant. Prove that your construction yields a knot invariant either combinatorially or by proving that it is equivalent to a topological construction.

   Hint: To derive the crossing condition, you might want to think about what the *topological* definition should be (regardless of how you prove invariance in the end).
3. Prove that $\pi_1(S^3 \setminus T(p,q)) \simeq \langle x, y \mid x^p = y^q \rangle$.

   Hint: Think of S^3 as the union of two solid tori glued along the torus that $T(p,q)$ lies on.
4. Suppose that a diagram D of a knot K has s Seifert circles (i.e. when you do Seifert's algorithm, you end up with s circles) and c crossings. Prove that

$$g(K) \leq \frac{1}{2}(1 + c - s),$$

   and give an example of a diagram for which this estimate is not sharp.