

Math 336 HW #7

Due 5:00 PM Tuesday, April 6

Presentations:

1. Let X be a CW -complex and let $A \subset X$ be a subcomplex (you should define this!). Define the relative cellular homology $H_*^{\text{cell}}(X, A)$ and prove directly that there is a long exact sequence

$$\cdots \rightarrow H_k^{\text{cell}}(A) \rightarrow H_k^{\text{cell}}(X) \rightarrow H_k^{\text{cell}}(X, A) \rightarrow \cdots.$$

2. Prove that the tricoloring number of a knot is an invariant using the Reidemeister moves.

Problems:

1. Hatcher p. 155 #1.
2. Hatcher p. 157 #18. Don't try to define the cellular chain maps geometrically; just define an analogue of the map $\hat{\partial}$ that we used in class, prove that your analogue satisfies $\hat{\partial}^2 = 0$, and declare it to be the cellular boundary map. The obvious purpose of this problem is to get you to understand the proof in class that cellular and singular homology coincide.
3. Hatcher p. 157 #19.
4. Suppose that X is a finite CW -complex with c_n n -cells. Define its Euler characteristic by $\chi(X) = \sum_n (-1)^n c_n$. As you proved for the Euler characteristic of a Δ -complex, the Euler characteristic of a CW -complex is independent of the CW -structure on X since $\chi(X) = \sum_n (-1)^n \text{rank } H_n(X)$ (see Hatcher p. 146 if you'd like). Using this definition, do Hatcher p. 157 #22 and #23. For the latter, note that we can *define* the genus ("number of holes") of an orientable surface F by $g(F) = 2 - 2\chi(F)$.
5. Show that any knot K may be unknotted by changing some subset of the crossings of its diagram D_K .