Math 336 HW #5

Due 5:00 PM Tuesday, March 2

Presentations:

1. Prove the Brouwer Fixed Point Theorem in n dimensions by proving the No Retract Lemma, i.e. that there does not exist a retract $r : D^n \to S^{n-1}$. You proved this for $n = 1, 2$ last semester.

2. Prove that $\mathbb{R}^n \not\cong \mathbb{R}^m$ for $n \neq m$. You proved this for $n = 1, 2$ last semester.

Problems:

1. Hatcher p. 132 #18.

2. Hatcher p. 133 #26. Briefly describe what went wrong. You may use the results of Example 1.25 without proof, but you must cite them precisely and correctly. Be careful with this problem!

3. Let X be the cylinder $S^1 \times [0, 1]$.
   (a) Compute the "local homology groups" $H_n(X, X \setminus x)$ at all points $x \in X$.
   (b) Suppose $f : X \to X$ is a homeomorphism. Can a boundary point be mapped to an interior point? NOTE: This is a simple case of an important general principle! Compare this with Class Presentation #2.

4. Let $A \subset B \subset X$. Prove that the relative homology groups $H_*(B, A)$, $H_*(X, A)$ and $H_*(X, B)$ fit into a long exact sequence:

$$\cdots \to H_n(B, A) \to H_n(X, A) \to H_n(X, B) \xrightarrow{\partial_*} H_{n-1}(B, A) \to \cdots .$$

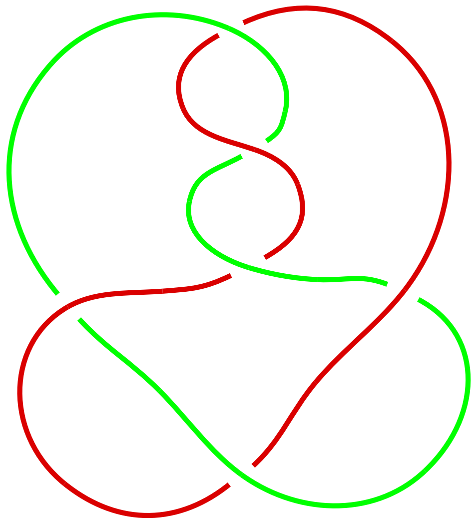
5. (a) Suppose that $0 \to A \xrightarrow{i} B \xrightarrow{j} C \to 0$ is a short exact sequence. Prove that if either
      i. There is a homomorphism $p : B \to A$ such that $p \circ i$ is the identity on A, or
      ii. There is a homomorphism $s : C \to B$ such that $j \circ s$ is the identity on C

   then $B \simeq A \oplus C$.

   (b) *Without using simplicial homology*, compute $H_*(S^n \vee S^m)$ for $n, m > 0$.

   NOTE: If you find it easier, you may compute the *reduced* homology in this problem. A hint for the second part: consider the relationship between the projection $q : S^n \vee S^m \to S^n \vee S^m / S^m = S^n$ and the inclusion $j : S^n \to S^n \vee S^m$.

BONUS Are the following two curves linked? A convincing "proof by picture" that references homology suffices.



This is worth an additional point, I suppose.