Math 336 HW #4

Due 5:00 PM Tuesday, February 23

Presentations:

1. Prove that if chain maps $\phi, \psi : A_* \to B_*$ are chain homotopic, then $\phi_* = \psi_*$.

2. Using notation from class, show that

$$\Delta^n \times I = [v_0, \ldots, v_n, w_n] \cup [v_0, \ldots, v_{n-1}, w_{n-1}, w_n] \cup \cdots \cup [v_0, w_0, \ldots, w_n].$$

   HINT: Use the "barycentric" coordinates in Hatcher, p. 103 to organize your proof.

Problems:

1. Suppose that $f : X \to Y$ is a continuous map with $f(x_0) = y_0$. Show that the following diagram commutes:

$$\begin{array}{ccc} \pi_1(X, x_0) & \xrightarrow{h} & H_1(X) \\ \downarrow{\scriptstyle f_*} & & \downarrow{\scriptstyle f_*} \\ \pi_1(Y, y_0) & \xrightarrow{h} & H_1(Y) \end{array}$$

   Here, h is the Hurewicz homomorphism, the f_* on the left is the induced map *on the fundamental group* and the f_* on the right is the induced map *on homology.*

2. If $p : \tilde{X} \to X$ is a covering map, is it true that $p_* : H_1(\tilde{X}) \to H_1(X)$ is injective? You can use simplicial homology in computing your examples.

3. Let X be $\mathbb{R}^2$ with n distinct points removed. Compute $H_0(X)$ and $H_1(X)$.

4. (a) Define a relation $\sim$ on chain maps $\phi, \psi : A_* \to B_*$ by $\phi \sim \psi$ if there is a chain homotopy between ϕ and ψ. Prove that $\sim$ is an equivalence relation.

   (b) We say that $\phi : A_* \to B_*$ is a **chain equivalence** if there exists a $\psi : B_* \to A_*$ such that $\phi\psi \sim id_B$ and $\psi\phi \sim id_A$. Define a relation on chain complexes by $A_* \sim B_*$ if there exists a chain equivalence $\phi : A_* \to B_*$. Prove that this is an equivalence relation. If $A_* \sim B_*$, what can you say about the relationship between $H_*(A)$ and $H_*(B)$?