Math 336 HW #2

Due 5:00 PM Tuesday, February 9

Presentations:

1. Prove that if $f : K \to K'$ is a Δ-map, then it induces a *continuous* map $|f| : |K| \to |K'|$.
2. Prove the second case of the proof that $f_\#$ is a chain map; that is, prove that if $f_\# \Delta_\alpha = 0$, then $\partial' f_\# \Delta_\alpha = f_\# \partial \Delta_\alpha$.

Problems:

1. Hatcher p. 131 Problem 8.
2. Hatcher p. 131 Problem 9.
3. Given a Δ-complex K, we can define the *cone of* K to be the Δ-complex CK defined by adding the following simplices to K: for every n-simplex $[v_0, \ldots, v_n]$ in K, add a new $(n+1)$-simplex $[v', v'_0, \ldots, v'_n]$. Also add in a new 0-simplex $[v']$. To get the identifications, identify the face $[v'_0, \ldots, v'_n]$ with $[v_0, \ldots, v_n]$ and the face $[v', \ldots, \widehat{v'_i}, \ldots]$ with the new n-simplex derived from the face $[v_0, \ldots, \widehat{v_i}, \ldots]$ of $[v_0, \ldots, v_n]$.
   (a) Explicitly enumerate the simplices and identifications for the cone on a Δ-complex K with 2 simplices of maximal dimension. Draw pictures!
   (b) Compute the simplicial homology of CK for any Δ-complex K.
4. Prove the classification of finitely generated abelian groups using Smith Normal Form and the fact that every subgroup of a finitely generated free abelian group is also a finitely generated free abelian group. The form of the classification theorem that you should prove is that every finitely generated abelian group is isomorphic to exactly one of the groups

$$\mathbb{Z}^n \oplus \mathbb{Z}/d_1 \oplus \cdots \oplus \mathbb{Z}/d_k,$$

   where $d_1 | d_2 | \cdots | d_k$.
5. (a) Find Δ-complexes K and K' such that $|K| \simeq S^2$, $|K'| \simeq \mathbb{R}P^2$, and there exists a 2-to-1 Δ-map $f : K \to K'$.
   (b) Find Δ-complexes K and K' such that $|K| \simeq S^2 \simeq |K'|$ and a Δ-map $g : K \to K'$ such that g_* is multiplication by n in dimension 2 (thinking of g_* as a map $g_* : \mathbb{Z} \to \mathbb{Z}$).