Math 336 HW #1

Due 5:00 PM Thursday, January 28

Presentations:

1. Let K be a Δ-complex. Prove that $U \subset |K|$ is open if and only if the intersection of U with each simplex in K is open. (*Hint:* part of this exercise is finding good notation. Once you do, it should be straightforward.)

2. Compute the homology groups of Δ-complex structures on the cylinder and on the Möbius band. What homology class does one boundary component of the cylinder represent? What homology class does the boundary of the Möbious band represent?

Problems:

1. Determine all topologically distinct Δ-complexes that can be constructed from a single triangle. When showing that two objects are the same it's fine to use (well-explained) pictures, but you need a rigorous argument to show that two are different.

2. Hatcher p. 131 Problem 2. A picture proof is fine.

3. Here's one of my favorite chain complexes, which you've actually seen before. Let

$$\begin{aligned}\Omega_0(\mathbb{R}^3) &= \Omega_3(\mathbb{R}^3) = C^\infty(\mathbb{R}^3, \mathbb{R})\\ \Omega_1(\mathbb{R}^3) &= \Omega_2(\mathbb{R}^3) = C^\infty(\mathbb{R}^3, \mathbb{R}^3)\\ \Omega_i(\mathbb{R}^3) &= 0 \text{ for } i \notin \{0, 1, 2, 3\}.\end{aligned}$$

   i.e., the zeroth and third groups are the infinitely-differentiable functions on $\mathbb{R}^3$, the first and second groups are the infinitely-differentiable vector fields on $\mathbb{R}^3$, and all other groups are zero. Also, let

$$\begin{aligned}\partial_3 &= \nabla \text{ (gradient)}\\ \partial_2 &= \nabla \times \text{ (curl)}\\ \partial_1 &= \nabla \cdot \text{ (divergence)}\end{aligned}$$

   Prove that (Ω_*, ∂) is a chain complex (explain why each ∂ is a group homomorphism and then prove that $\partial^2 = 0$) and compute $H_3(\Omega_*, \partial)$.

4. Hatcher p. 131 Problem 4.

5. Hatcher p. 131 Problem 5.