Math 3200: Day 7

Problems to present: 6, 20, 24(a-c) & (d-f)

Last time you (hopefully) spent quite a bit of time thinking about conditionals & biconditionals & working at truth tables.

Let's do a simple example:

Ex: Consider the compound statement "If I bring my umbrella, then I'll bring my umbrella if it rains."

How to translate into symbols?

Can let U: "I bring my umbrella" & R: "It rains."

Then the sentence is "If U then (U if R)" or $U \Rightarrow (R \Rightarrow U)$.

Truth table:

U	R	$R \Rightarrow U$	$U \Rightarrow (R \Rightarrow U)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

This sentence is an example of a tautology: a compound statement that is necessarily true regardless of the truth values of its component pieces.

Since $A \Rightarrow B$ is equivalent to $(\sim A) \vee B$, the statement $U \Rightarrow (R \Rightarrow U)$ could also be written as

$(\sim U) \vee (R \Rightarrow U)$ ⇝ "Either I won't bring my umbrella, or I'll bring my umbrella if it rains"

or even $(\sim U) \vee ((\sim R) \vee U)$ ⇝ "Either I won't bring my umbrella, or either it won't rain or I'll bring my umbrella,"

which makes it more clear that this is a tautology.

Another fun tautology: $A \Rightarrow A$, as we can see from the truth table:

A	$A \Rightarrow A$
T	T
F	T

The opposite of a tautology is a contradiction, which is a statement which is never true, regardless of the truth values of its components.

Q: Example of a logical contradiction?

A: e.g. $\sim(A \Rightarrow A)$, $A \wedge (\sim A)$, $(A \Leftrightarrow B) \wedge (A \wedge (\sim B))$, etc.

A statement which is neither a tautology nor a contradiction is sometimes called a contingent statement, since its truth value is contingent on the truth of its constituents.