Math 3200: Day 40

Exam 3 grade breakdown

A	≥ 18
B	14–17
C	10–13
D/F	≤ 9

Comments: Counterexamples, bad proofs, injectivity/surjectivity

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Last time I proved the following claim:

Thm: For any set A, $|A| < |\mathcal{P}(A)|$.

Proof: Clearly the map $f: A \to \mathcal{P}(A)$ given by $f(x) = \{x\}$ is injective, so $|A| \leq |\mathcal{P}(A)|$. Hence, we need only show that there is no surjective map $g: A \to \mathcal{P}(A)$.

I'll prove this by showing that, for any $g: A \to \mathcal{P}(A)$, there is some element of $\mathcal{P}(A)$ (i.e. some subset of A) that is not in the image of g.

The idea is similar to Russell's Paradox.

Let $B = \{x \in A : x \notin g(x)\}$. Then clearly $B \subseteq A$, meaning $B \in \mathcal{P}(A)$, since B consists of elements of A.

I claim that there is no $a \in A$ so that $B = g(a)$. Suppose there were such an a. Then $B = g(a)$. Now, is $a \in B$ or not? If $a \in B$, then $a \in g(a)$... but that contradicts the definition of B.

Alternatively, if $a \notin B$, then $a \notin g(a)$, so $a \in B$!

Since one or the other must be true, we conclude that there is no $a \in A$ so that $B = g(a)$.

But then g is not surjective. $\blacksquare$