Math 3200: Day 4

Problems to present: 10(b), 16, 24, 50

Logic:

Def: A *statement* is a declarative sentence that is either true or false.

Ex: "There are 37 people in this room"

"It is raining"

"44 has 2 prime factors"

"Every even integer greater than 2 is the sum of 2 primes."

Non-Ex: "$\sqrt{\xi}=2$" (What ξ?)

"Let $A=\{1,2,3\}$"

"Math is fun!"

Suppose P & Q are statements. Then we can associate the following *truth table*, which records the possible combinations of truth values:

P	Q
T	T
T	F
F	T
F	F

Def: A *open sentence* is a declarative sentence containing variables which becomes a statement when values from the appropriate domain are substituted for the variable(s).

Ex: The ex. from before, "$\sqrt{\xi}=2$" is an open sentence for any reasonable choice of domain for ξ (i.e. $\mathbb{N}$ or $\mathbb{Z}$ or $\mathbb{Q}$ or $\mathbb{R}$, ...)

but not if we let the domain be, say, the people in this room.

When the domain is $\mathbb{N}$, we see that "$\sqrt{\xi}=2$" is true when $\xi=4$ & false otherwise.

Ex: $P(x,y)$: "the point (x,y) is 2 units from the origin" (equivalently, $P(x)$: $\sqrt{x^2+y^2}=2$ or $P(x)$: $x^2+y^2=4$) is an open sentence when the domain of x is $\mathbb{R}$ & the domain of y is also $\mathbb{R}$. In this case, $P(2,0)$, $P(0,-2)$ & $P(\sqrt{2},\sqrt{2})$ are all true, while $P(1,1)$, $P(0,0)$ & $P(3,4)$ are all false.

Logical Operators: Not, Or, And

Negation: If P: 44 has 2 prime factors, then the negation of P is:

$\sim P$: It is not the case that 44 has 2 prime factors

Q: Examples?