Math 3200: Day 34

Prop: Suppose A & B are finite sets with $|A|=|B|$. A map $f:A\to B$ is injective iff it is surjective.

Proof: ($\Rightarrow$) Suppose f is injective. Then $|B|=|A|=|f(A)|$, so since $f(A)\subseteq B$, it must be the case that $f(A)=B$, so f is surjective.

($\Leftarrow$) Suppose f is surjective. Then $|f(A)|=|B|=|A|$, so f must be injective. ∎

Note: This proposition is not true for infinite sets. Whatever cardinality means for infinite sets, surely $|\mathbb{Z}|=|\mathbb{Z}|$. But

$f:\mathbb{Z}\to\mathbb{Z}$ given by $f(x)=2x$ is injective & not surjective

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Suppose you have $f:A\to B$ & $g:B\to C$. Then how can we get from A to C?

We compose the two functions: for each $a\in A$, $f(a)\in B$, so we can apply g to it to get $g(f(a))\in C$.

In other words, the composition of the functions $f:A\to B$ & $g:B\to C$ is the function

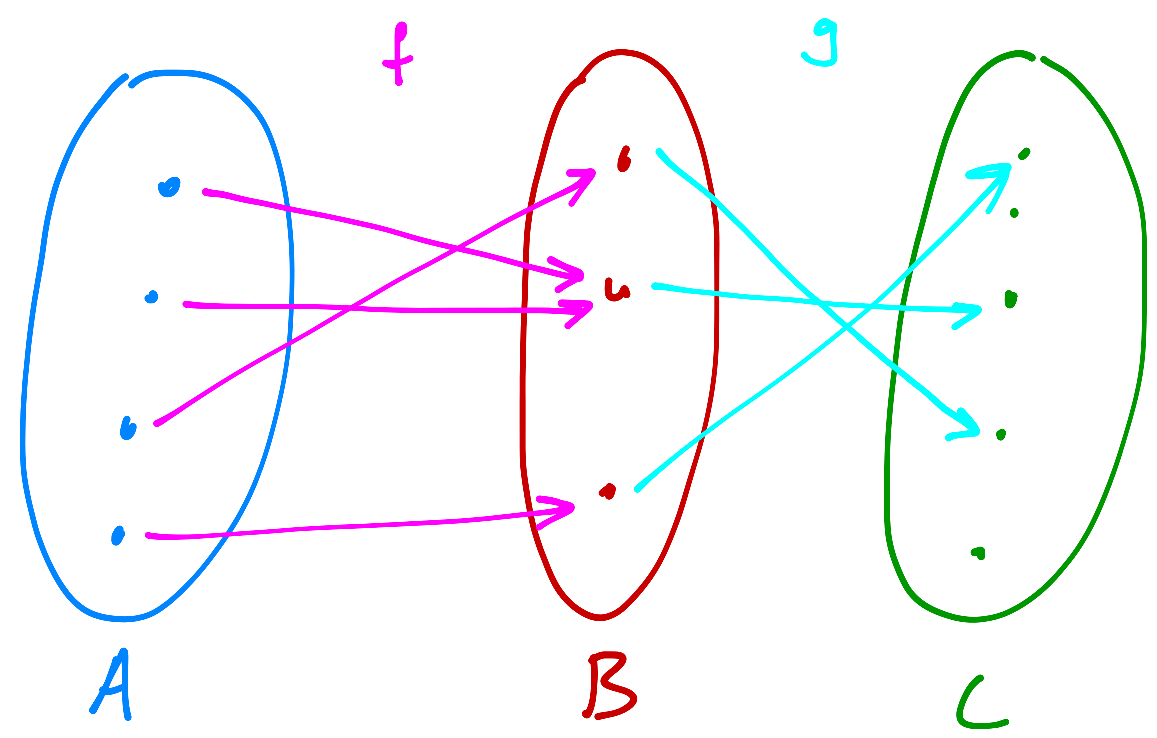
$$g\circ f: A\to C \quad \text{defined by} \quad (g\circ f)(a)=g(f(a)).$$

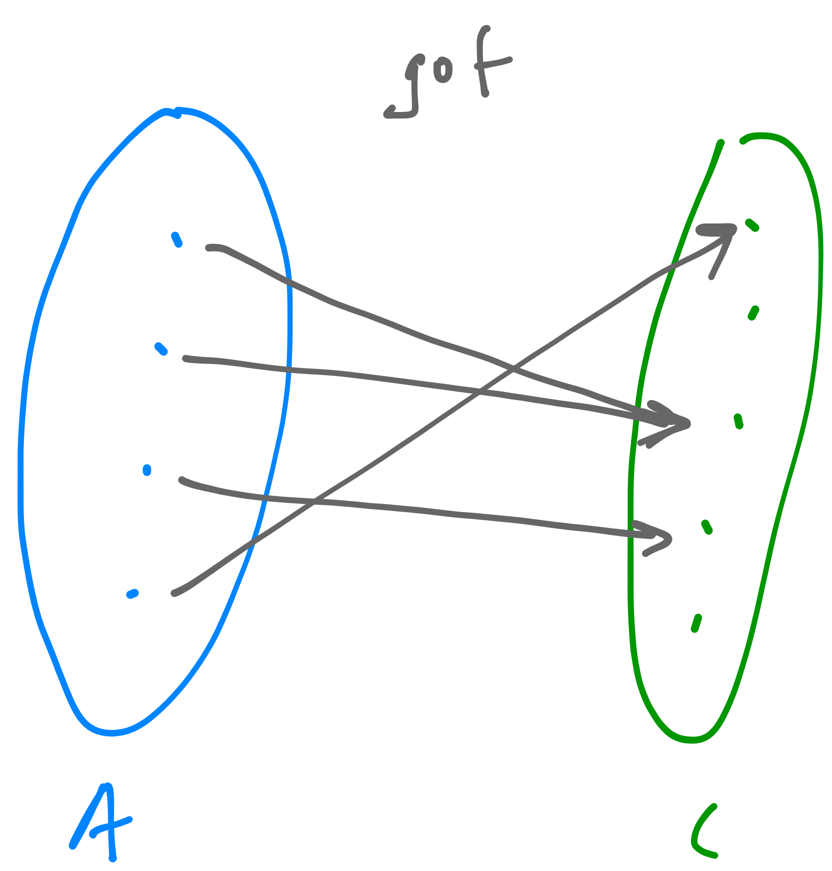
The order is important: if you do g first, you're in C, so applying f makes no sense.

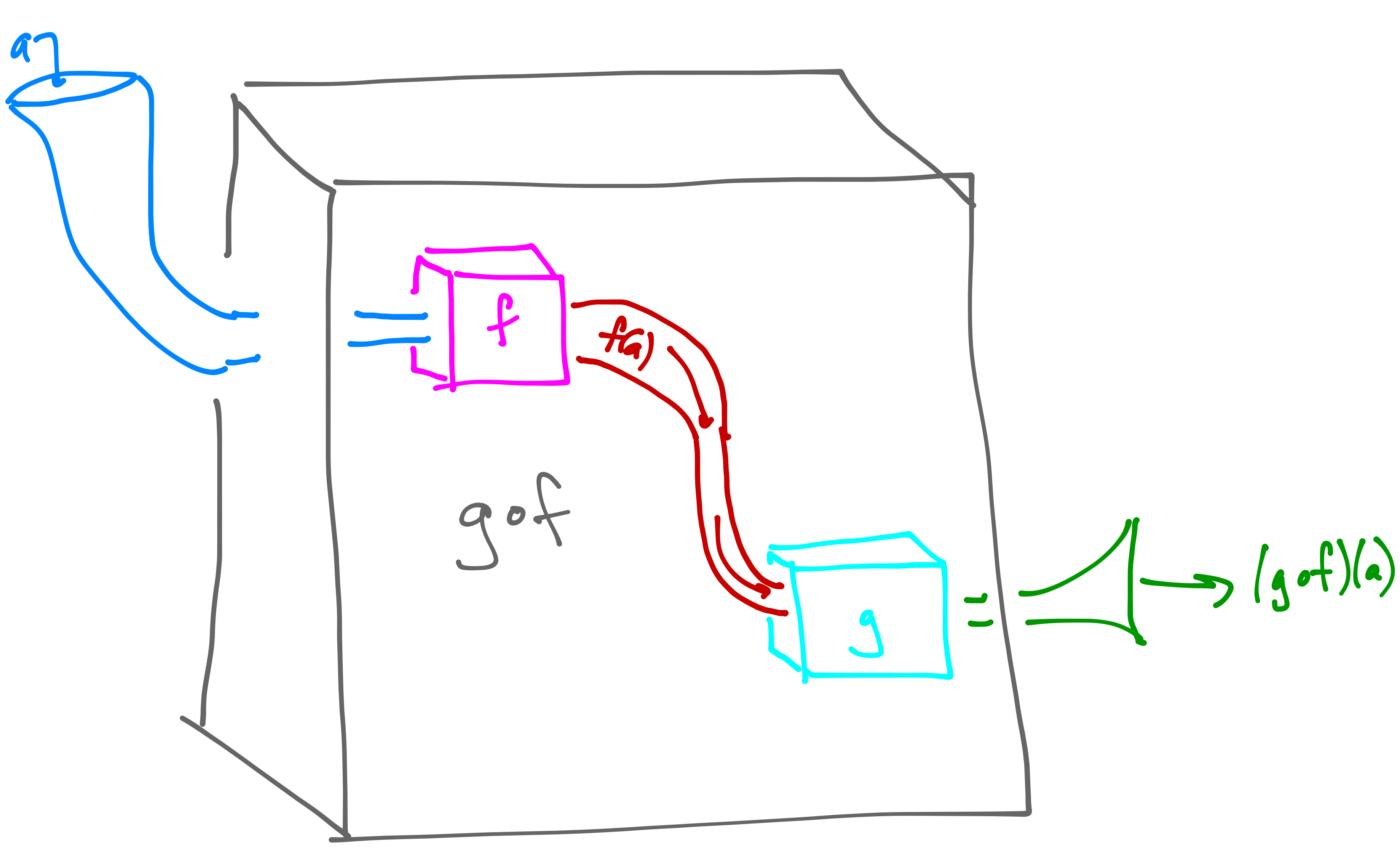
For example, if $A:\mathbb{R}\to\mathbb{R}_{\geq 0}$ is given by $A(t)=$ altitude of plane @ time t & $T:\mathbb{R}_{\geq 0}\to\mathbb{R}$ is given by $T(h)=$ temperature @ altitude h

then $T\circ A:\mathbb{R}\to\mathbb{R}$ gives the external temp. of the plane at time t, whereas $A\circ T$ is meaningless.

Cartoons:







Unless A=C, you can't hook the machines up in the opposite order.