Math 3200: Day 25

Grade Ranges for Exam 2

A	≥ 19
B	15-18
C	11-14
D/F	≤ 10

Comments: #3: If you're trying to prove something by contradiction, it's very important to explicitly assume the negation of the statement you're trying to prove (and to make sure you correctly negate that statement)

#5: $P(n)$ is not equal to $\frac{1}{1\cdot 2}+\dots+\frac{1}{n(n+1)}$ ... it is the statement "$\frac{1}{1\cdot 2}+\dots+\frac{1}{n(n+1)} = 1-\frac{1}{n+1}$"

Relations:

In mathematics, it's usually the relationships between objects that are interesting. For example, we're interested in when $x=y$ or $x<y$ or when $a|b$ or when $a\equiv b \pmod n$ or when $A\subseteq \mathcal{P}(B)$ or when line ℓ_1 is parallel to line ℓ_2 or when triangle T_1 is similar to triangle T_2 or when rectangle R_1 has the same area as rectangle R_2 or when functions f & g have the same derivative or when $f(2)=g(2)$ or when f & g have the same Taylor series or when topological space X is homeomorphic to topological space Y.

Clearly all of these concepts embody some common essential feature, so we need to formalize that.

Def: If A & B are sets, then a relation R from A to B is a subset $R\subseteq A\times B$.

Ex: If $A=B=\mathbb{N}$, then the relation "is the same parity as" is the subset of $\mathbb{N}\times\mathbb{N}$ given by

$$\{(1,1),(1,3),\dots,(2,2),(2,4),\dots,(3,1),(3,3),\dots,\dots\}$$

Notation: If R is a relation from A to B and $(a,b)\in R$, it's common to write aRb.

Ex: The $=$ relation on $\mathbb{Z}$ is given by the following subset of $\mathbb{Z}\times\mathbb{Z}$:

$$\{\ldots,(-1,-1),(0,0),(1,1),(2,2),\ldots\}$$

obviously, we usually write $x=y$ rather than $(x,y)\in =$.

Ex: Consider $A=\{1,2,3\}$ & $B=\{2,3,4\}$. Then the relation $\leq$ is given by

$$\{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4)\}$$

The inverse relation of a relation R is the relation R^{-1} from B to A given by:

$$\{(b,a)\in B\times A : (a,b)\in R\}$$

So in the above example, the inverse relation is the $\geq$ relation from B to A given by

$$\{(2,1),(3,1),(4,1),(2,2),(3,2),(4,2),(3,3),(4,3)\}\subseteq B\times A.$$

Def: A relation R on a set A is symmetric if for all $(x,y)\in R$, we also have $(y,x)\in R$.

Ex: · Equality is symmetric: if $x=y$, then $y=x$.

· $\leq$ is not symmetric: $5\leq 6$, but $6\nleq 5$.