Math 3200: Day 2

Let $A=\{1,2,3\}$, $B=\{1,2,4\}$, $U=\{1,2,3,4,5\}$. Then $A\subseteq U$ & $B\subseteq U$, but $A\not\subseteq B$ & $B\not\subseteq A$.

Def: The power set $\mathcal{P}(A)$ of a set A is the set of all subsets of A.

Ex: If $A=\{1,2,3\}$, then $\mathcal{P}(A)=\{\emptyset,\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$

If $D=\{1,\{1\},\{\{1\}\}\}$, then $\mathcal{P}(D)=\{\emptyset,\{1\},\{\{1\}\},\{\{\{1\}\}\},\{1,\{1\}\},\{1,\{\{1\}\}\},\{\{1\},\{\{1\}\}\},\{1,\{1\},\{\{1\}\}\}\}$

In particular, notice that $X\in\mathcal{P}(X)$ for any set X.

Def: The cardinality of a finite set A, denoted $|A|$, is the number of elements it contains.

Ex: W/ A & D as above, $|A|=|D|=3$. Also, $|\mathcal{P}(A)|=|\mathcal{P}(D)|=8$.

Conjecture as to relationship b/w $|A|$ & $|\mathcal{P}(A)|$ in general?

(A: $|\mathcal{P}(A)|=2^{|A|}$)

A deep & interesting question which we will return to later: how to make sense of the cardinality of an infinite set?

Set operations by example:

Union: In the above example, the union of A & B is $A\cup B=\{1,2,3,4\}$.

Intersection: $A\cap B=\{1,2\}$

Complement: $\overline{A}=\{4,5\}$, $\overline{B}=\{3,5\}$.

Q: How to define these rigorously?

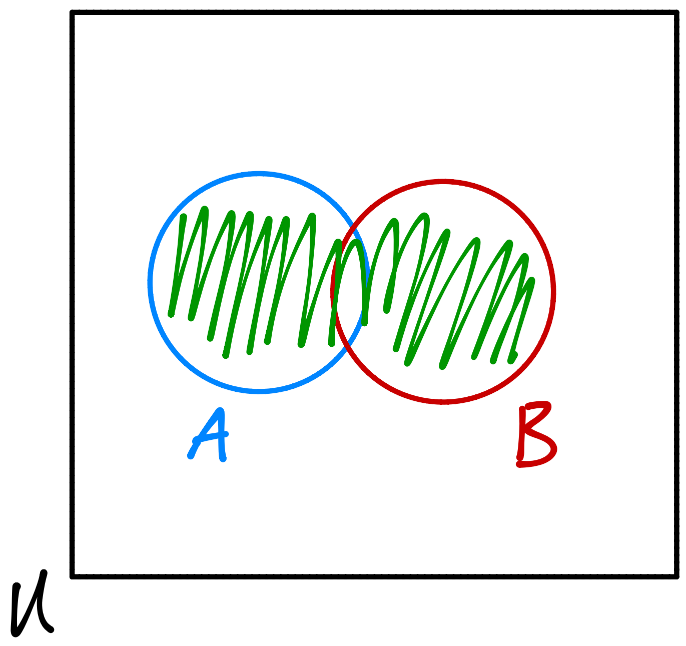
A: Def: Let A & B be subsets of some universal set U. Then the union of A & B consists of all elements of U which are contained in A or B (or both). In other words, $x\in A\cup B$ if & only if $x\in A$ or $x\in B$.

On the other hand, $x\in A\cap B$, the intersection of A & B, if & only if $x\in A$ and $x\in B$.

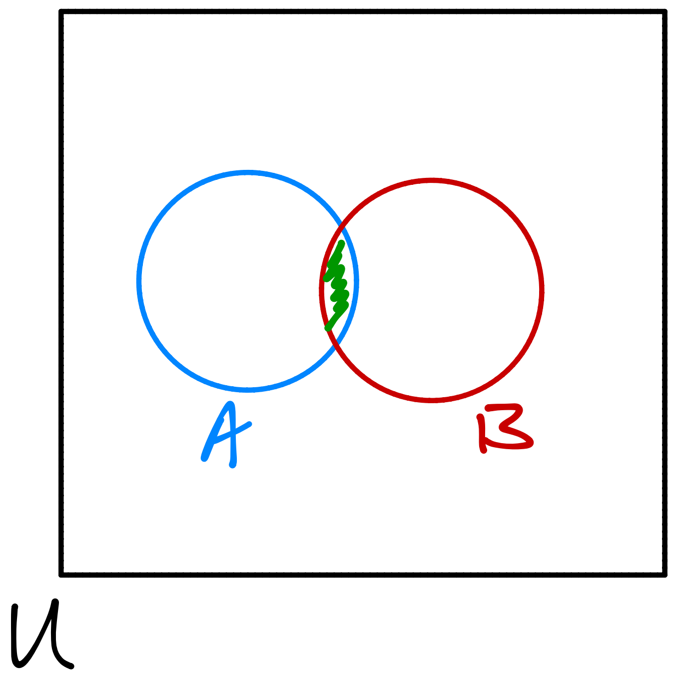
Finally, the complement of the set A (relative to the universe U) consists of all $x\in U$ such that $x\notin A$.

Venn diagrams:

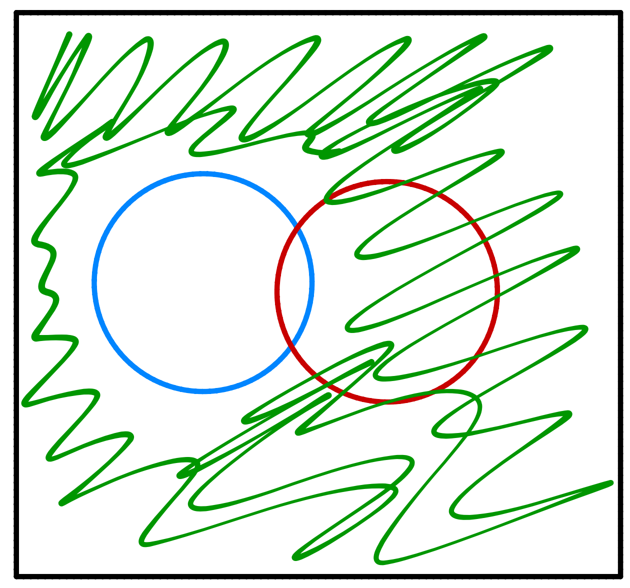
$A \cup B$



$A \cap B$



$\bar{A}$



Def: The *difference* of sets A & B, denoted $A-B$ or $A\setminus B$, is the set

$$A-B=\{x\in A : x\notin B\}.$$

Ex: · $\mathbb{R}-\mathbb{Q}=\{x : x \text{ is irrational}\}$

· $\mathbb{Q}-(1,2)=\,?$ Notice that $(1,2)\not\subseteq\mathbb{Q}$, but we can still make sense of this: it's the set of rational #s which are ≤ 1 or ≥ 2. So $\mathbb{Q}-(1,2)=\{x\in\mathbb{Q} : x\leq 1 \text{ or } x\geq 2\}=\mathbb{Q}\cap\big((-\infty,1]\cup[2,+\infty)\big)$

Let's think a little bit more carefully about what it means to say that 2 sets are equal.

In other words, if I say that two sets A & B are equal: (i) what does that really mean?

(ii) How would I prove it?

A: (i) $A=B$ if & only if A & B contain exactly the same elements. In other words, for each $a\in A$, it must be the case that $a\in B$ and for each $b\in B$, we have $b\in A$.

(ii) Notice that the first condition above implies $A\subseteq B$, & the second implies $B\subseteq A$. The most common strategy for showing $A=B$ is to show that $A\subseteq B$ and $B\subseteq A$.

This seems overly elaborate, but it's completely necessary if we want to prove something like, say, (one of) DeMorgan's Laws:

$$(A-B)\cap(A-C)=A-(B\cup C) \quad \text{for any sets } A,B,C.$$