Math 3200: Day 10

Really, the proofs we did last time were too easy. Usually a statement is neither trivially nor vacuously true, so we have to work a bit harder

(Look back at the statements of the Goldbach Conjecture & Fermat's Last Theorem. These are clearly neither trivial nor vacuous)

Prop: If n is an odd integer, then n^2+n+1 is also odd.

Proof: Let $n \in \mathbb{Z}$ be odd, meaning that $n = 2m+1$ for some $m \in \mathbb{Z}$. Then

$$n^2+n+1 = (2m+1)^2 + (2m+1) + 1 = (4m^2+4m+1) + (2m+1) + 1$$
$$= 4m^2+6m+3.$$

Now, $4m^2+6m+3 = (4m^2+6m+2)+1 = 2(2m^2+3m+1)+1$ is odd, so we see that n^2+n+1 is odd, as desired. ∎

Note: I only have to assume the antecedent is true & show that the consequent follows. If the antecedent is false then the statement is automatically true.

Prop: If a is an odd integer, then $3a-4$ is even.

Proof: Let $a \in \mathbb{Z}$ be odd, meaning that $a = 2n+1$ for some $n \in \mathbb{Z}$. Then

$$3a-4 = 3(2n+1)-4 = 6n+3-7 = 6n-4 = 2(3n-2)$$

is even, as desired. ∎

In the above examples, there was nothing tricky. We used basic definitions, did some simple algebraic manipulation, & the result fell right out.