Math 3200: Day 1

Q: What is mathematics?

A: Wikipedia says that "Mathematics is the abstract study of topics such as quantity (numbers), structure, space, and change."

G.H. Hardy said that "A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas."

Whether we want to embrace Wikipedia or Hardy or something else, the cultural fact is that mathematics is the deductive study of abstract things, and that mathematics has its own language & customs that we have to get used to.

The basic building blocks of mathematics are definitions, lemmas, propositions, theorems, corollaries, and most importantly examples.

Throughout this course we'll have to get comfortable with using the language of mathematics & to thinking in a different way than you're probably used to.

The fundamental abstract objects in mathematics are sets. In fact (most of) mathematics can be translated purely into statements about sets, though this is rarely how people think & not something we'll really get into.

Nonetheless, it will be essential to develop some fluency with sets, so that's where we start:

Def: A set is a (unordered) collection of objects. The members of a set are called elements of the set. If a is an element of a set A, we can write this as $a \in A$.

Ex: ① The set $\{1,2,3\}$ consists of the positive integers 1, 2, and 3. Order doesn't matter, so $\{1,2,3\}$ is the same as $\{2,1,3\}$ or $\{3,2,1\}$, etc.

② $\{\text{Erin}, \text{John}\}$ is a set consisting of the words "Erin" & "John". We could also consider the set consisting of two people, one named Erin & one named John, which we might or might not choose to denote the same way.

③ $\{x : x \text{ is an integer solution to } x^2-2x=0\}$ consists of the integer solutions of $x^2-2x=0$. This is equal to the set $\{0,2\}$.

④ $\{x : x \text{ is a real soln to } x^2+5=0\}$ is empty: it contains no elements.

Df: The empty set, denoted $\emptyset$, is the (unique) set with no elements.

Special Notation: $\emptyset$ empty set

$\mathbb{N}$ natural numbers, i.e., $\{1,2,3,\dots\}$

$\mathbb{Z}$ integers, i.e., $\{\dots,-2,-1,0,1,2,\dots\}$

$\mathbb{Q}$ rational numbers

$\mathbb{R}$ real numbers

$\mathbb{C}$ complex numbers.

Clearly there are relationships between these sets. For example, all integers are also rational numbers, so all the elements of $\mathbb{Z}$ are also elements of $\mathbb{Q}$. In other words, if $x \in \mathbb{Z}$, then $x \in \mathbb{Q}$.

Df: If A & B are sets so that $x \in A$ implies that $x \in B$, then A is a subset of B, denoted $A \subseteq B$.

Ex: $\emptyset \subseteq \mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$.

Ex: Let $A = \{1,2,3\}$, $B = \{1,2,4\}$, and $C = \{1,2,3,4,5\}$

Then notice $A \subseteq C$ & $B \subseteq C$. However, $A \nsubseteq B$ & $B \nsubseteq A$. The sets A & B are incomparable.