Math 3200 Exam #1 Practice Problems

1. Let $S = \{1, 2, 3\}$ and let $A = \{1\}$. What is the cardinality of the set $\mathcal{P}(S) - \mathcal{P}(A)$?

2. Prove that if $x \in \mathbb{R}$ and $x > 2$, then $x^2 - x > 2$. (*Hint: it's often helpful to get* 0 *on the right side of inequalities.*)

3. For each $n \in \mathbb{N}$, let $A_n = [0, \frac{3}{n})$.

   (a) What is $\bigcup_{n \in \mathbb{N}} A_n$?

   (b) What is $\bigcap_{n \in \mathbb{N}} A_n$?

4. Give an example of an infinite set $S \subset \mathcal{P}(\mathbb{Q})$ so that $S \neq \mathcal{P}(\mathbb{Q})$.

5. Re-write each of the following statements in symbols. Please explain any notation you introduce (for example, if you want $E(x)$ to stand for the open sentence "x is even", then please say so explicitly).

   (a) All positive integers are prime.

   (b) Some positive real numbers are irrational.

   (c) All integers are perfect squares.

6. For each of the statements in problem 5, say whether the statement is true or false. Explain your answer.

7. Consider the following statement: For $n \in \mathbb{N}$, if $3n^2 - 2n - 1$ is even, then n is odd. Either prove the statement or give a counterexample which shows that it is false.

8. Suppose that $x, y \in \mathbb{Z}$ are both divisible by 3. Prove that for any integers a and b, the integer $ax+by+1$ is not divisible by 3.

9. Let $O = \{2n - 1 : n \in \mathbb{N}\}$ and let $T = \{3m : m \in \mathbb{N}\}$. Write the set $O \cap T$ in two different ways.

10. For each of the following statements, convert it to symbols and then say whether the statement is a tautology, a contradiction, or neither and explain why.

    (a) Either I won't bring my umbrella or if it rains I'll bring my umbrella.

    (b) No matter whether it rains or not, I'm bringing my umbrella.