

Math 318 Final Projects

Spring 2011

Listed below are some ideas for possible final projects, but I would encourage you to pursue your own ideas and I would be happy to discuss the suitability of particular ideas.

Guidelines

I have two main types of projects in mind, though certainly various combinations and variants are also possible. The first is more theory-oriented, in which the primary goal is (likely) to present an interesting theorem and its proof, providing the necessary definitions and presenting worked examples of both key definitions and theorems. The second is more applied, with less of an emphasis on theory (though some definitions and theorems will almost certainly be necessary) and more on how a particular idea or package of ideas from analysis is used to solve some type of problem of real-world interest; examples would again be essential, along with detailed computations implementing the application.

Regardless of the type of project, the final report should be well-written, with complete sentences, good grammar, a notable lack of typos and formatting issues, etc. The report should definitely not just be a long string of equations with no explanations. In fact, part of good writing is knowing what hairy details to leave out: lengthy computations whose details are not central to the main ideas of the project can (and probably should) be omitted (of course, you should say clearly that you are doing so).

How long should the report be? The true (and thereby useless) answer is “As long as it needs to be”. In practice, I’m envisioning something on the order of 8–12 pages, though you shouldn’t feel constrained by the upper limit if your work is rich with pictures and/or examples. A shorter paper which accomplishes something substantial would also be fine.

You are encouraged but not required to use \LaTeX , a mathematical typesetting tool that we require for math senior theses. It has a learning curve (which I and Curtis will help you with), but once you’ve mastered the basics, it tends to be the fastest way to produce professional-looking mathematics, and leaves all other tools in the dust when it comes to cross-references.

You should start thinking about possible thesis topics now: talk to me and Curtis, visit the library, search Google, check out sections of the text we haven’t gotten to yet (or won’t ever get to).

Possible project descriptions

Dedekind cuts

At the beginning of Math 317, we defined \mathbb{R} as a complete ordered field. This is fine, but leaves open the question of whether there is such a thing, and whether it is unique. There is another approach that is (sort of) more direct, but a bit mind-bending, called Dedekind cuts; see Abbott §8.4

e^x and its properties

We will talk about how to define e^x as a series early in the semester and demonstrate that it equals its own derivative. However, is it really the same as some number e raised to the power x ? To understand that, you need to do a little more, including proving the key properties $e^{x+y} = e^x e^y$, and that, in turn, requires learning about multiplying series, and some facts about double-summations that we skipped (see Abbott §2.8). In addition, one could define the inverse function of e^x (i.e., $\ln x$) and derive a few of its key properties.

$\sin x$, $\cos x$ and their properties

Define these functions as series; use series-of-functions methods to prove they are continuous, differentiable, what their derivatives equal; then, show that \cos and \sin really means what you saw them to mean in trigonometry; using the complex versions of the series definitions, demonstrate the famous equation $e^{ix} = \cos x + i \sin x$. Possible starting point: Marsden & Hoffman §5.4

Multivariable integration: Fubini's Theorem

Define integration in \mathbb{R}^2 ; Fubini's Theorem: how to write an \mathbb{R}^2 -integral as a double integral, when can you swap the order of integrals and when can you not. Possible starting point: Marsden & Hoffman §8.1–8.3, 9.1–9.2

Multivariable integration: Change of Variables Theorem

Define integration in \mathbb{R}^2 ; Change of Variables Theorem: how to change variables within an integral; applications to polar, spherical coordinates. Possible starting point: §Marsden & Hoffman 8.1–8.3, 9.3

Multivariable derivative theorems

Inverse and Implicit Function Theorems; Applications (Domain-Straightening Theorem, Range-Straightening Theorem, Morse Lemma) prove you can assign “curved coordinates” in which functions have some simple form; illustration of these geometric coordinatizations on the computer. Possible starting point: Marsden & Hoffman Chapter 7

The δ -function and distributions

The δ -function as traditionally pseudo-defined in physics; more rigorous definition of δ -function as a type of object called a “distribution” and/or via integration with respect to a “concentrated measure” as we’ll see in class; use of distributions to extend application of Fourier transforms to periodic functions (connecting back to Fourier series). Possible starting point: Marsden & Hoffman §8.7

Integration in Path Space

We’ll be developing a measure on \mathbb{R} and using it to define an integration over \mathbb{R} . One can do the same thing in “path space”, i.e., the set of certain functions. This then allows one to integrate over path space. This has applications to Brownian motion and the Black-Scholes equation in mathematical finance.

Discrete Dynamical Systems

Fixed points; contraction principle; definition of chaos; “period-3 implies chaos”, fractals. Possible starting point: Davidson & Donsig Chapter 11

Fixed point theorems

Contraction mapping theorem to prove existence of fixed point; related fixed point theorems from topology: Hairy Ball Theorem and Brouwer Fixed Point Theorem; role of fixed point theorems in game theory. Possible starting point: Davidson & Donsig §11.1, plus outside material

Fourier series

We will cover some of the basic theory of Fourier series near the end of the semester, including orthogonal polynomials, and the specific “Fourier basis”, but this is a topic which is both broad and deep, so there are plenty of possibilities for projects related to Fourier series, including discussing convergence theorems, or the application of Fourier series to solve certain PDEs, or their use in frequency-analyzing some time series of interest. Possible starting point: Abbott §8.3, or other analysis texts.

Existence of solutions to differential equations

Contraction mapping theorem; application to existence and uniqueness theorem for ordinary differential equations; examples when solutions do or do not exist, are or are not unique; use of contraction mapping theorem to compute solution of differential equation in special cases. Possible starting point: Davidson & Donsig Ch. 12

Fourier transforms

What is a Fourier transform (continuous analogue of Fourier series); properties of Fourier transform; computing Fourier transform efficiently (e.g., “fast Fourier transform”); applications to signal analysis or compression. Possible Starting point: §Marsden & Hoffman 10.8, plus outside material

Laplace transforms and ordinary differential equations

Definition of Laplace transform (variant of Fourier transform); properties of Laplace transform; application to solving certain ordinary differential equations. Possible starting point: Marsden & Hoffman §10.8 and outside material

Functional analysis in quantum mechanics

Differential operators as functions on function space; Spectral theory for eigenvalues of differential operators; application to spectrum of hydrogen atom. Possible starting point: Cohen.

Calculus of vector fields

Physical interpretations (current distributions, fluid/plasma flows); vector fields on curved spaces; connections to the topology of the domain; the Helmholtz decomposition. Possible starting point: see me.

Differential forms

The “right” way to do multivariable calculus. Connections to (multi-)linear algebra; coordinate independence; integration and Stokes’ Theorem; Hodge decomposition. Possible starting point: Spivak.