

Math 318 Exam #2

Due 3:00 PM Friday, April 1.

Name: _____

You may refer to your notes, your text, your homework, or the homework solutions. Do not discuss the questions with anyone except me.

There is no time limit, and the exam is due at 3:00 PM on Friday, April 1. If you plan to ask me questions about course content, please do not look at the problems until after you do so.

Please do not sign the following until the examination is completed.

I accept full responsibility under the Haverford Honor Code for my conduct on this examination.

Signed: _____

1. Suppose that $A \subset [0, 1]$ such that $m^*(A) = 1$. Show that A is dense in $[0, 1]$.
2. Show that every subset of $[0, 1]$ with positive Lebesgue measure contains a pair of distinct points whose distance apart is a rational number.

(If you're struggling with this, I will give partial credit for proving that every subset of $[0, 1]$ with positive Lebesgue measure contains a pair of points whose distance apart is an *irrational* number.)

3. Suppose f is continuous a.e. on a bounded measurable set A . Show that f is measurable on A .

(Notice this has as an immediate corollary that every Riemann integrable function is Lebesgue integrable.)

4. Given positive integers i and k , define the function

$$f_{i,k}(x) := \begin{cases} 1 & \text{if } \frac{i-1}{k} < x \leq \frac{i}{k} \\ 0 & \text{else} \end{cases}$$

on the interval $(0,1]$. Show that the sequence

$$f_{1,1}, f_{1,2}, f_{2,2}, f_{1,3}, f_{2,3}, f_{3,3}, \dots, f_{1,n}, f_{2,n}, \dots, f_{n,n}, \dots$$

converges in measure to zero (see Problem #3 from HW 7 for the definition), but does not actually converge at any point in $(0, 1]$.

5. Suppose A and B are bounded, measurable sets such that $m(A \triangle B) = 0$. If f is Lebesgue integrable on $A \cup B$, show that

$$\int_A f dm = \int_B f dm.$$

Feel free to use Theorem 24.2.

(Note: this result combined with Problem #3 from HW 6 implies that we can always compute Lebesgue integrals over Borel sets.)