Math 317 Exam #2 Practice Problems

1. For each of the following statements, say whether it is true or false. If the statement is true, explain why; if it is false, give a counterexample.

   (a) The equation $2x^{17} - 4x^{12} + 1 = 5x^{23} + 10x^8 - 2$ has a solution in the interval $[0, 1]$.

   (b) Every series of the form $\sum(-1)^n a_n$ converges.

   (c) The series $\sum_{n=1}^{\infty} \frac{\cos^n x}{n^2}$ converges uniformly on all of $\mathbb{R}$.

2. Give examples of each of the following, or explain why no such example exists.

   (a) A uniformly continuous function on the interval $(0, 1)$.

   (b) A continuous function with domain the interval $[0, 1]$ and range the interval $(0, 1)$.

   (c) A sequence (f_n) of continuous functions which converges uniformly to a continuous function f on some set S.

   (d) A power series with interval of convergence $(-2, 2)$.

3. (a) Give a rigorous definition of

$$\lim_{x \to +\infty} f(x) = L.$$

   (b) Using your definition from part (a), show that

$$\lim_{x \to +\infty} \frac{1}{x^2} = 0.$$

4. Determine the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^n}{\sqrt{n}} x^n.$$

5. Let $f : \mathbb{R} \to \mathbb{R}$ be an increasing function defined on all of $\mathbb{R}$. Prove that $\lim_{x \to c^+} f(x)$ exists for every $c \in \mathbb{R}$.