Math 317 Exam #1 Practice Problems

1. For each of the following statements, say whether it is true or false. If the statement is true give a brief explanation why (this does not need to be a proof); if it is false, give a counterexample.

    (a) Every bounded set contains its supremum.

    (b) A decreasing sequence is always bounded.

    (c) Every Cauchy sequence is bounded.

    (d) Every sequence contains a convergent subsequence.

2. Give examples of each of the following, or explain why no such example exists (this explanation does not need to be a proof, but you should give a clear and convincing explanation).

    (a) An alternating sequence that converges. (An *alternating sequence* is one whose terms alternate between positive and negative.)

    (b) A bounded, divergent sequence.

3. Determine the following numbers (don't worry about proofs; just find the answer any way you like).

    (a) The infimum of the set
    $$\{x \in \mathbb{R} : -1 < 2x - x^2\}.$$

    (b) $\lim_{n\to\infty} a_n$ where the terms of the sequence (a_n) are given by
    $$a_n = \frac{2n-3}{5n^2+4}.$$

4. A number t is called a *cluster point* of the sequence (s_n) if for every $\epsilon > 0$ there are infinitely many values of n with $|s_n - t| < \epsilon$.

    (a) Prove that if there is a subsequence of (s_n) that converges to t, then t is a cluster point of (s_n).

    (b) True or false: every bounded sequence has a cluster point. If the statement is true, explain why. If it is false, find an example of a bounded sequence with no cluster point.