Math 317 Exam #2

Due 3:00 PM Friday, November 12.

Name: ____

You may refer to your notes, your text, your homework, or the homework solutions. Do not discuss the questions with anyone except me.

There is no time limit, and the exam is due at 3:00 PM on Friday, November 12. If you plan to ask me questions about course content, please do not look at the problems until after you do so.

Please do not sign the following until the examination is completed.

I accept full responsibility under the Haverford Honor Code for my conduct on this examination.

Signed: ____

1. Suppose $S \subseteq \mathbb{R}$ is connected and contains more than 1 point. Show that every element of S is a limit point of S.

2. Let $A \subseteq \mathbb{R}$ be uncountable. Show that A has a limit point.

   (*Hint: consider bounded subsets of* A.)

3. Suppose $A, B \subseteq \mathbb{R}$ with A compact, B closed, and $A \cap B = \emptyset$.

   (a) Show that there exists $\epsilon > 0$ such that $|a - b| > \epsilon$ for all $a \in A$ and $b \in B$.

   (b) Does the result in (a) still hold when A is closed but not necessarily compact? (B is still closed.)

4. Suppose $A \subseteq \mathbb{R}$ and $a \in A$. Then a is called an *interior point* of A if there exists some $\epsilon > 0$ such that $V_\epsilon(a) \subseteq A$. The *interior* of A is defined to be the set $\mathring{A}$ consisting exactly of the interior points of A.

   (a) If $A \subseteq \mathbb{R}$, do A and $\overline{A}$ always have the same interior? Do A and $\mathring{A}$ always have the same closure?

   (b) Prove that $\mathring{A}$ is always open. What is the complement of $\mathring{A}$?

5. (a) Give a rigorous definition (in the style of Definition 4.2.1) of

$$\lim_{x \to +\infty} f(x) = L.$$

   (b) Using your definition from part (a), show that

$$\lim_{x \to +\infty} \frac{1}{x^2} = 0.$$