Math 2260 HW #8
Due 10:10 AM Friday, March 2

Reading: Hass §8.6–8.7

Problems: Do the assignment "HW8" on WebWork. In addition, write up solutions to the following problems and hand in your solutions in class on Friday.

1. Suppose a and b are two fixed positive numbers. Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

   [*Hint: you might want to find the area of the half of the ellipse above the x-axis and then double it.*]

2. Evaluate the integral

$$\int \frac{e^{4t} + 2e^{2t} - e^t}{e^{2t} + 1} \, dt.$$

3. Sociologists sometimes use the phrase "social diffusion" to describe the way information spreads through a population. The information could be anything: a scientific breakthrough, news of a natural disaster, literacy, etc. In a sufficiently large population, the number of people x who have the information is treated as a differentiable function of time t, and the rate of diffusion, dx/dt, is assumed to be proportional to the number of people who have the information times the number of people who do not. This leads to the equation

$$\frac{dx}{dt} = kx(N - x),$$

   where N is the number of people in the population. The constant k determines how fast the information spreads: you would expect k to be relatively large for, say, celebrity gossip, which is assimilated almost instantly, whereas it would be quite small for information that only disseminates very slowly (like how to compute integrals!).

   Suppose t is in days, $k = 1/250$, and two people start a rumor at time $t = 0$ in a population of $N = 1000$ people.

   (a) Find x as a function of t. [*Hint: Logarithm identities are your friends!*]

   (b) When will half the population have heard the rumor? (This is when the rumor will be spreading the fastest.)

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