Math 2260 Written HW #1 Solutions

1. Evaluate the definite integral

$$\int_4^9 \frac{2}{\sqrt{x}} \, dx.$$

   Answer: We can re-write this integral as

$$\int_4^9 2x^{-1/2} \, dx,$$

   which we can then integrate using the Fundamental Theorem of Calculus and the Power Rule:

$$
\begin{aligned}
\int_4^9 2x^{-1/2} \, dx &= \left[2\frac{x^{1/2}}{1/2} \right]_4^9 \\
&= \left[4\sqrt{x} \right]_4^9 \\
&= 4\sqrt{9} - 4\sqrt{4} \\
&= 12 - 8 \\
&= 4.
\end{aligned}
$$

   Therefore, we can conclude that $\int_4^9 2x^{-1/2} \, dx = 4$.

2. Evaluate the indefinite integral

$$\int \frac{\cos(x)}{\sin^5(x)} \, dx.$$

   Answer: I want to compute this integral using u-substitution. Let $u = \sin x$. Then $du = \cos x \, dx$, and so the above integral is equal to

$$\int \frac{du}{u^5} = \int u^{-5} du = \frac{u^{-4}}{-4} + C = -\frac{1}{4u^4} + C = -\frac{1}{4\sin^4 x} + C.$$